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Supplement of

The effect of rainfall variability on Nitrogen dynamics in a small agricultural catchment

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Table S1. The parameters for the N source zone. The parameters were adjustable and calibrated
(referring to *Yang et al.* [2022]).

Parameter	Process	Adjustable range	Reference	Value of best fit
k_a	Mineralization	0~0.7	Van Meter et al. [2017]	0.0111 day ⁻¹
k_p	Mineralization	0~0.7	Van Meter et al. [2017]	0.0008 day ⁻¹
<i>p</i> 1	Plant uptake	60~160	Van Meter et al. [2017]	160 kg·ha ⁻¹
<i>p</i> 2	Plant uptake	1~10		10 kg·ha ⁻¹
<i>p</i> 3	Plant uptake	1~60		34 day
λ_s	Denitrification in soil	0~0.7	Nguyen et al. [2021]	0.0007 day ⁻¹
k_l	Leaching	1~1000		18.8888 day ⁻¹
q_{ref}	Leaching	1e ⁻⁴ ~1		0.01 m·day ⁻¹
λ	Denitrification in water	1e ⁵ ~1e ⁻¹	Nguyen et al. [2021]	0.0088 day ⁻¹

S1 Stochastic rainfall generator

16

Daily rainfall time series with specific climatic characteristics are generated by the stochastic rainfall generator in three steps [Robinson & Sivapalan, 1997]. First, determining a series of alternating storm durations t_r and inter-storm periods t_b over the simulation time. It is hypothesized that t_r and t_b are independent of each other and vary seasonally. Each t_r and t_b as random variables are sampled from the exponential (and shifted exponential) probability density functions (pdf) regarding monthly mean storm duration γ and mean inter-storm period δ , respectively:

24
$$f_{T_r}(t_r|\gamma) = \frac{1}{\gamma} exp\left(-\frac{t_r}{\gamma}\right) \quad t_r > 0$$
 (S1)

25
$$f_{T_b}(t_b|\delta) = \frac{1}{\delta - \varepsilon} exp\left(-\frac{t_b - \varepsilon}{\delta - \varepsilon}\right) \quad t_b > \varepsilon$$
 (S2)

- where the shift ε is the specified minimum dry period of 24 hours. Both γ and δ are
- 27 assumed as sinusoidally with time of year τ as follow:

28
$$\gamma = \gamma_s + \alpha_{\gamma} \sin\left\{\frac{2\pi}{\omega_h}(\tau - \tau_{\gamma})\right\}$$
 (S3)

29
$$\delta = \delta_s + \alpha_\delta \sin\left\{\frac{2\pi}{\omega_h}(\tau - \tau_\delta)\right\}$$
 (S4)

- 30 where γ_s and δ_s are the seasonally averaged storm duration and inter-storm period,
- 31 respectively. α_{γ} and α_{δ} are the amplitudes of the seasonal variations in γ and δ .
- 32 τ_{γ} and τ_{δ} are seasonal phase shifts of t_r and t_b . ω_h is the total number of time
- units in a year. If τ , τ_{γ} , and τ_{δ} are in days, ω_h is equal to 365 days. Regional
- 34 climates are determined by multiple factors, such as atmosphere, geography, hydrology,
- 35 and biology). As a result, τ_{γ} and τ_{δ} are definite and steady for a certain area.
- However, climate change may reflect in the variation of γ_s , δ_s , α_{γ} and α_{δ} , which

- 37 alters the average level of storm duration and inter-storm period and their distribution
- 38 during a year.
- 39 Second, determining the average rainfall intensity i of each storm. Average rainfall
- 40 intensity i is set as a random variable stochastically dependent on storm duration t_r ,
- which means that i and t_r follow the joint probability density function, $f_{I,T_r}(i,t_r|\gamma)$.
- In order to characterize the dependence between i and t_r , it is assumed that the
- 43 conditional statistics of $E[i|t_r]$ and $CV^2[i|t_r]$ are power functions of t_r .

44
$$E[i|t_r] = a_1 t_r^{b_1}$$
 (S5)

45
$$CV^2[i|t_r] = a_2 t_r^{b_2}$$
 (S6)

- The dependence is seasonally varying, which embodies in parameters a_1 and b_1 . The
- 47 values of b_1 and b_2 represent the extent of the dependence of $E[i|t_r]$ and
- 48 $CV^2[i|t_r]$ on t_r . The value of a_2 represents the correlation between the mean of
- 49 annual maximum rainfall intensity and t_r . The occurrence of extreme climate events
- may impact the average rainfall intensity.
- 51 The conditional distribution of i given t_r follows the gamma distribution:

52
$$f_I(i|t_r) = \frac{\lambda}{\Gamma(\kappa)} (\lambda i)^{\kappa - 1} exp(-\lambda i)$$
 (S7)

- According to (S5) and (S6), the gamma parameters κ and λ are estimated as the
- 54 functions of t_r as follows:

$$\kappa = \frac{t_r^{-b_2}}{a_2} \tag{S8}$$

$$56 \lambda = \frac{t_r^{-b_1 - b_2}}{a_1 a_2} (S9)$$

Combined with (S1), (S3), (S7), (S8), and (S9), the joint pdf of i and t_r is derived as

58 follows:

59
$$f_{I,T_r}(i,t_r|\gamma) = f_I(i|t_r)f_{T_r}(t_r|\gamma)$$
 (S10)

- 60 Third, constructing within-storm rainfall intensity variations using a normalized mass
- 61 curve (Huff, 1967; Chow et al., 1988):

62
$$H(t^*) = \frac{1}{it_r} \int_0^t i(t') dt'$$
 (S11)

- Where $t^* = t/t_r$ ($0 \le t^* \le 1$), i is the average rainfall intensity for the single storm,
- and i(t') is the instantaneous rainfall intensity at time t' within the certain storm
- 65 $(0 \le t' \le t_r)$. In order to generate realistic within-storm rainfall intensity patterns, a
- stochastic model capable of producing normalized mass curves satisfying the statistical
- 67 characteristic was developed to replace the equation above (referring to Robinson and
- 68 Sivapalan, [1997], section 3.4.2).

In order to elucidate the effect of inter-annual rainfall variability on N dynamic, 100 stochastic rainfall time series of WY, NY, and DY (EDY) were generated by the stochastic rainfall generator (Figure S1) to substitute for the rainfall data in the simulation period.

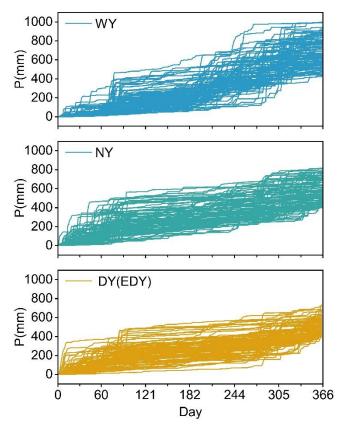


Figure S1. Daily cumulative values of 100 stochastic rainfall time series generated by the stochastic rainfall generator for WY, NY, and DY (EDY).

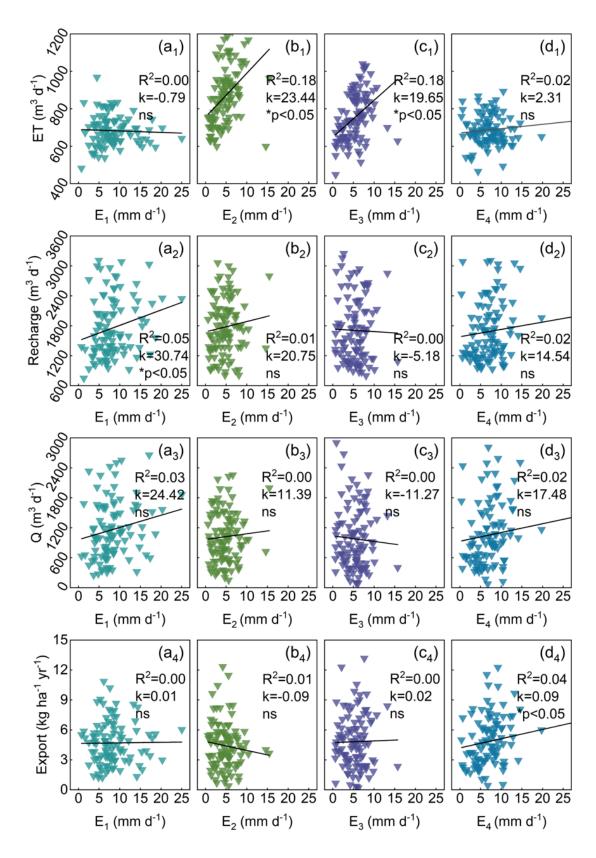


Figure S2. The responses of actual evapotranspiration (ET), recharge for groundwater, discharge (Q), and N export to the average rainfall intensity of the four seasons (E_1 - E_4). The determination coefficients (R^2) between E_1 - E_4 and ET, recharge, Q, and export are listed. The sign and magnitude of the slopes (k) in these linear relationships denote the direction and

- 81 intensity of the response of N dynamics to the variations in average rainfall intensity. Asterisks
- 82 indicate the significance of the regression slopes (p < 0.05); ns denotes non-significant
- 83 relationships (p ≥ 0.05).