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Enhancing urban pluvial flood modeling through graph reconstruction of incomplete sewer networks

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S1 Slope estimation using multiple points

Given a path consisting of K points $p = \{v_1, ..., v_K\}$ with corresponding elevation values $z = \{z_1, ..., z_K\}$, its slope can be derived from the estimated coefficient of ordinary least squares regression through the coordinates (v_r, z_r):

$$i_p = \frac{\sum_{r=1}^{K}(d_r - \mu_d)(z_i - \mu_z)}{\sum_{r=1}^{K}(d_r - \mu_d)^2} = \frac{\sum_{r=1}^{K} d_r z_r - K\mu_d\mu_z}{\sum_{i=1}^{K} d_r^2 - K\mu_d^2} \tag{1}$$

where d_r is the distance from v_1 to v_r along the path ($d_1 = 0$), μ_d and μ_z is the mean value of $\{d_r\}_{r=1,..,K}$ and $\{z_r\}_{r=1,..,K}$.

Eq. 1 can re-written by using z_r as pivot variables:

$$\begin{aligned} i_p &= \frac{\sum_{r=1}^{K} d_r z_r - K\mu_d\mu_z}{\sum_{i=1}^{K} d_r^2 - K\mu_d^2} \\ &= \frac{K\sum_{r=1}^{K} d_r z_r - (\sum_{r=1}^{K} d_r)\cdot(\sum_{r=1}^{K} z_r)}{K\sum_{r=1}^{K} d_r^2 - (\sum_{r=1}^{K} d_r)^2} \\ &= \sum_{k=1}^{K} \frac{\left(K d_k - \sum_{r=1}^{K} d_r\right)}{K\sum_{r=1}^{K} d_r^2 - \left(\sum_{r=1}^{K} d_r\right)^2} z_k \end{aligned} \tag{2}$$

Thus, i_p can be expressed as a linear combination of z_r. When $K = 2$, i.e, only two points are involved in the slope estimation, we can simplify Eq. 2 into Eq. 3:

$$i_p = -\frac{1}{d_2} z_1 + \frac{1}{d_2} z_2 \tag{3}$$

S2 Bidirectional surface-sewer flow coupling scheme

In AUTOSHED, the surface flow and sewer flow are bidirectionally linked via the mass source term $Q_{\mathrm{1D-2D}}$ which can be parameterized as follows (Buttinger-Kreuzhuber et al., 2022):

$$Q^*_{\mathrm{1D-2D}} = \begin{cases} \frac{2}{3} c_w P \sqrt{2g} h_{\mathrm{2D}}^{1.5}, & H_{\mathrm{1D}} < z_{\mathrm{bc}} \\ c_{o,s} A \sqrt{2g} [h_{\mathrm{2D}} - (H_{\mathrm{1D}} - z_{\mathrm{bc}})]^{0.5}, & z_{\mathrm{bc}} \leq H_{\mathrm{1D}} < z_{\mathrm{bc}} + h_{\mathrm{2D}} - A/P \\ c_{w,s} P \sqrt{2g} h_{\mathrm{2D}} [h_{\mathrm{2D}} - (H_{\mathrm{1D}} - z_{\mathrm{bc}})]^{0.5}, & z_{\mathrm{bc}} + h_{\mathrm{2D}} - A/P \leq H_{\mathrm{1D}} < z_{\mathrm{bc}} + h_{\mathrm{2D}} \\ - c_{w,s} P \sqrt{2g} h_{\mathrm{2D}} [(H_{\mathrm{1D}} - z_{\mathrm{bc}}) - h_{\mathrm{2D}}]^{0.5}, & z_{\mathrm{bc}} + h_{\mathrm{2D}} \leq H_{\mathrm{1D}} < z_{\mathrm{bc}} + h_{\mathrm{2D}} + A/P \\ - c_{o,s} A \sqrt{2g} [(H_{\mathrm{1D}} - z_{\mathrm{bc}}) - h_{\mathrm{2D}}]^{0.5}, & H_{\mathrm{1D}} \geq z_{\mathrm{bc}} + h_{\mathrm{2D}} + A/P \end{cases} \quad (4)$$

Here we assume $Q^*_{\mathrm{1D-2D}}$ is positive when water flows from the TSU to the linked inlet. H_{1D}, h_{2D} and z_{bc} are the total hydraulic head at the inlet, the water depth at the TSU and the center bed elevation at the TSU, respectively; A and P are the inlet's area and perimeter, respectively; c_w, $c_{w,s}$ and $c_{o,s}$ are discharge coefficients for the free weir, the submerged weir, and the orifice equations, respectively. By default, AUTOSHED sets $c_w = 0.56$, $c_{w,s} = 0.11$ and $c_{o,s} = 0.2$ (Rubinato et al., 2017) and $A = 0.3375\ \mathrm{m}^2$, $P = 2.4\ \mathrm{m}$ assuming that each inlet is rectangular whose size is $0.45\ \mathrm{m} \times 0.75\ \mathrm{m}$ according to field investigation.

After examining the links between TSUs and inlets, we find that each TSU is linked with at most one inlet and thus have:

$$q_{\mathrm{sd}} = \frac{Q_{\mathrm{1D-2D}}}{A_{\mathrm{T}}} \quad (5)$$

where A_{T} is the total area of the TSU, and $Q_{\mathrm{1D-2D}}$ is the sewer drainage rate at the inlet derived from Eq. 4 and further constrained by the water availability of both the TSU and the inlet (Buttinger-Kreuzhuber et al., 2022):

$$Q_{\mathrm{1D-2D}} = \begin{cases} \min\left(\frac{h_{\mathrm{2D}} A_T}{\Delta t}, Q^*_{\mathrm{1D-2D}} \right), & H_{\mathrm{1D}} < z_{\mathrm{bc}} + h_{\mathrm{2D}} \\ \max\left(-\frac{V_{\mathrm{as}}}{\Delta t}, Q^*_{\mathrm{1D-2D}} \right), & H_{\mathrm{1D}} \geq z_{\mathrm{bc}} + h_{\mathrm{2D}} \end{cases} \quad (6)$$

where V_{as} is the water volume at the inlet assembly determined by A_{as} and H_{1D}; Δt is the coupling time step evaluated as the minimum value between the timesteps of the 1D and 2D solvers (Chen et al., 2018).

Notably, Eq. 5 and Eq. 6 can ensure strict mass balance during surface-sewer flow coupling since it is straightforward to prove:

- The total exchange mass is the same between the surface and sewer network

  $$q_{\mathrm{sd}} A_{\mathrm{T}} \Delta t = Q_{\mathrm{1D-2D}} \Delta t \quad (7)$$

- The total exchange mass does not exceed the available water in the surface when water flows from the TSU to the inlet, i.e., $H_{\mathrm{1D}} < z_{\mathrm{bc}} + h_{\mathrm{2D}}$,

  $$Q_{\mathrm{1D-2D}} \Delta t = \min\left(h_{\mathrm{2D}} A_T, Q^*_{\mathrm{1D-2D}} \right) \leq h_{\mathrm{2D}} A_T \quad (8)$$

- The total exchange mass does not exceed the available water in the sewer network when water flows from the inlet to the TSU, i.e., $H_{\mathrm{1D}} \geq z_{\mathrm{bc}} + h_{\mathrm{2D}}$,

  $$Q_{\mathrm{1D-2D}} \Delta t = \max\left(-V_{\mathrm{as}}, Q^*_{\mathrm{1D-2D}} \right) \geq -V_{\mathrm{as}} \quad (9)$$

In order to jointly solve surface-sewer flow equations coupled with Eq. 5, AUTOSHED utilizes the 1D routing portion of the Storm Water Management Model (SWMM) 5.1.015 (Rossman, 2015) and implements communication routines for 1D/2D state variables such as water depth and hydraulic head (Eq. 4) via interface functions by compiling SWMM as a dynamic link library (Leandro and Martins, 2016).

In addition to unregulated nodes and links, we also account for the controlling effects of pump stations and wastewater treatment plants using Type 4 pumps (whose flow rate changes linearly with the inlet node depth until the maximum pumping capacity is reached) and nodes with limited maximum inflows in SWMM, respectively (Rossman, 2015).

S3 Evaluation criteria for inundation area simulation

The Probability of Detection (POD), False Alarm Rate (FAR), Critical Success Index (CSI) and Bias (BIAS) are selected to evaluate the model performance in inundation area simulations (McGrath et al., 2018):

$$\mathrm{POD} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}} \tag{10}$$

$$\mathrm{FAR} = \frac{\mathrm{FP}}{\mathrm{TP}+\mathrm{FP}} \tag{11}$$

$$\mathrm{CSI} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}+\mathrm{FP}} \tag{12}$$

$$\mathrm{BIAS} = \frac{\mathrm{TP}+\mathrm{FP}}{\mathrm{TP}+\mathrm{FN}} \tag{13}$$

where TP (True Positive) represents areas correctly identified as inundated, FN (False Negative) represents areas incorrectly classified as not inundated, and FP (False Positive) represents areas incorrectly classified as inundated, with the results simulated by **F**ull-**S**ewer-**R**econstruction-with-controls (FSR) approach used as the reference. An area is considered inundated when its maximum depth exceeds 15 cm per local regulations.

S4 2-hour design rainfall scenarios in the city zone of Yinchuan

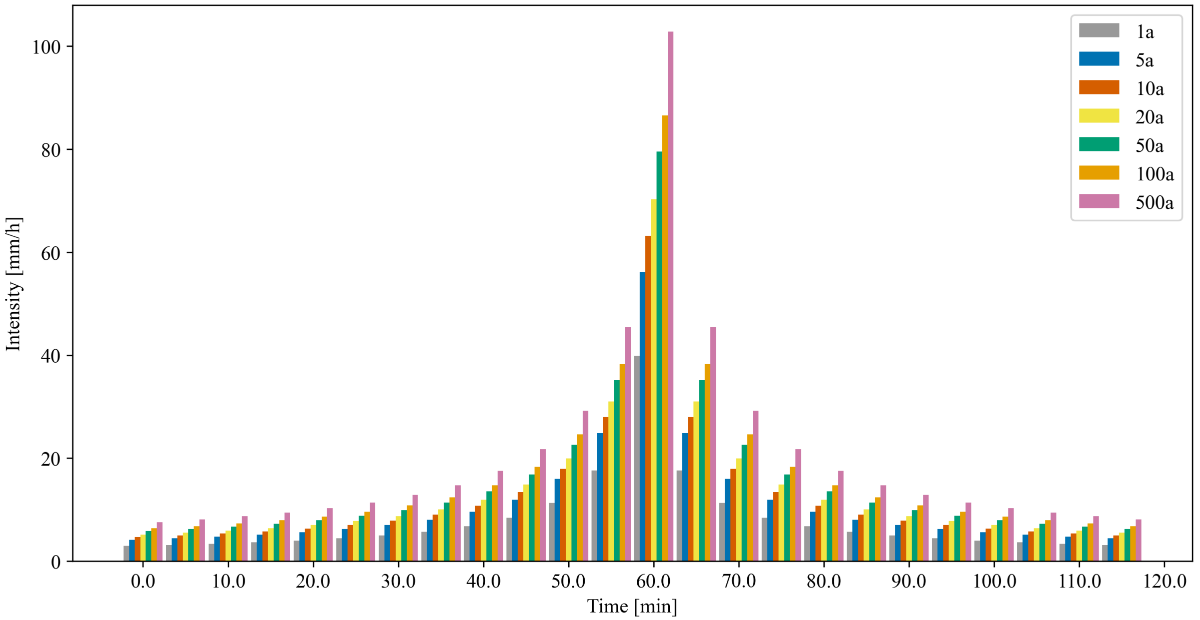

Figure S1. 2-hour design rainfall scenarios with varying return periods (1a, 5a, 10a, 20a, 50a, 100a and 500a).

S5 Simulation error of maximum inundation depth at inundated points using different reconstructed sewer hydraulic modules.

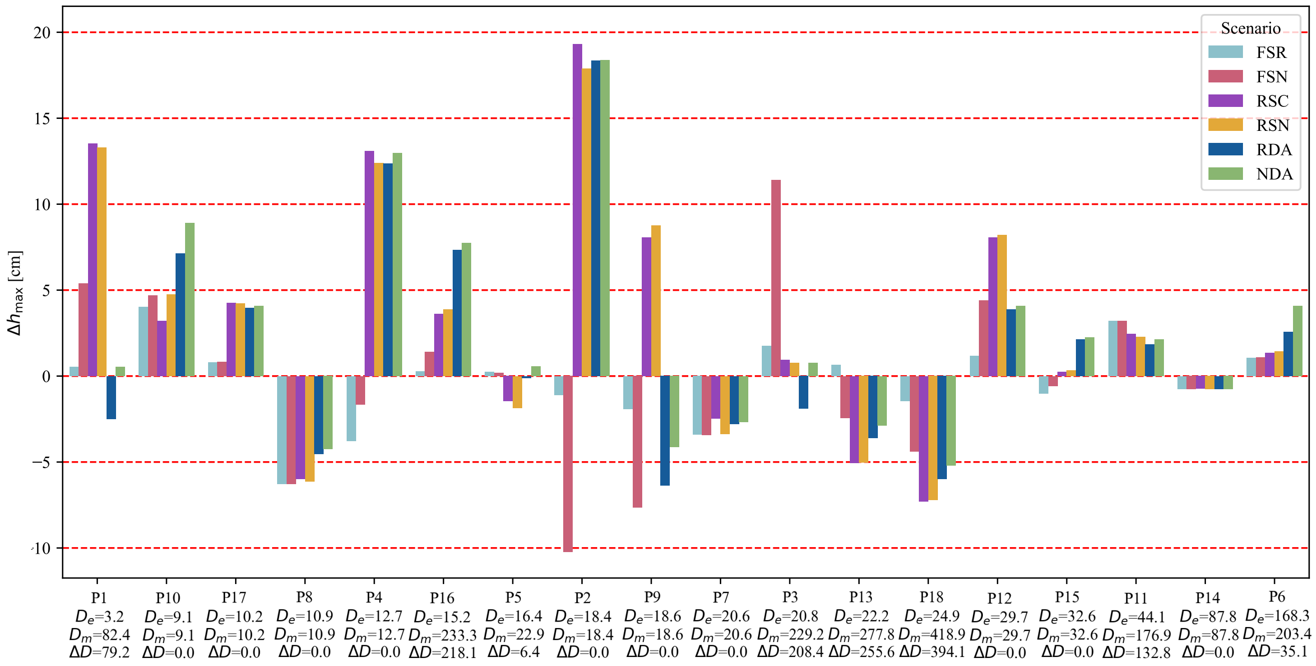

Figure S2. Simulation error of maximum inundation depth at inundated points where $\Delta h_{\max}$ stands for the simulated depth minus observed ones and D_f and D_s are the distances to the closest rainwater inlet of the complete sewer network used in the FSR, FSN approach and the simplified sewer network used in the RSN approach. $\Delta D = D_s - D_f$. The index of inundated points follow Fig. 8.

S6 Simulated outflow process error of maximum inundation depth at inundated points with different reconstructed sewer hydraulic modules

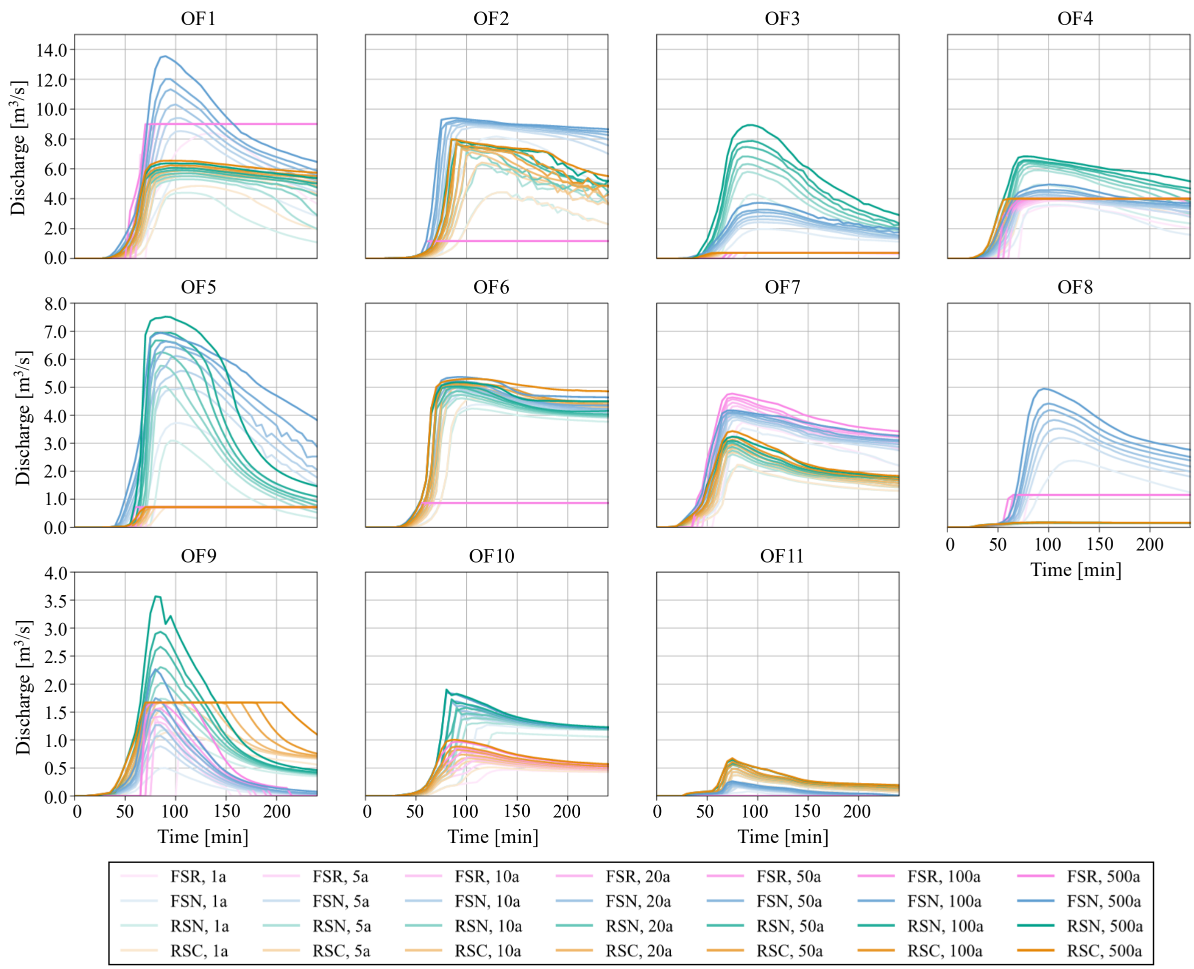

Figure S3. Comparison of simulated outflow processes under design rainfall scenarios using different reconstructed sewer hydraulic modules.

S7 Calibrated parameters of the AUTOSHED model

Table S1. Calibrated parameters of the AUTOSHED model.

Land use type	Impervious ratio $[\mathrm{m}^2/\mathrm{m}^2]$	Manning coefficient $[\mathrm{s/m}^{1/3}]$
tree	0.05	0.06
grassland	0.05	0.08
cropland	0.05	0.08
built-up	1.0	0.015
bareland	0.4	0.04
water	1.0	0.02

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