



Supplement of

***Thirsty Earth*: a game-based approach to interdisciplinary water resource education**

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S1 Thirsty Earth v0: Profit function and solutions

Players impersonate farmers choosing how much water to withdraw every year from a common groundwater irrigation resource in order to maximize their profit. Annual returns are proportional to the amount of water withdrawn, whereas annual costs are proportional to both the amount of water withdrawn and the depth of the water table. The latter is given by the total amount of water withdrawn by all players sharing. Specifically, following the *River Basin Game* (Hoekstra, 2012), annual profits for player i at period t are expressed as:

$$U_{it} = \alpha Q_{it} - Q_{it} \cdot \frac{C_{0,t} + (C_{0,t} + 1) + (C_{0,t} + 2) \dots + (C + Q_t - 1)}{Q_t}$$

$$= \alpha Q_{it} - Q_{it} \left(C_{0,t} + \frac{Q_t - 1}{2} \right)$$

where Q_{it} is the volume of water withdrawn by player i and $Q_t = \sum_{j \in [1, N]} Q_{jt}$ is the total amount of water withdrawn by the N players of community. Costs $C_{0,t}$ of the initial unit of water withdrawn by the community at period t are given as the difference between the volume of water in the aquifer before period 1 (S_0) and current water storage S_t at the beginning of period t :

$$C_{0,t} = S_0 - S_t$$

Current storage is determined by players' pumping decisions, a constant groundwater recharge R between each period, and groundwater discharge losses to streams that are proportional to groundwater storage and modeled as a linear reservoir with a recession constant k :

$$S_t = S_{t-1} - Q_{t-1} + R - S_{t-1}/k$$

Parameters by default in *Thirsty Earth* v0 are $\alpha = 50$, $k = 1.25$ and $R = 8 \cdot N$, $S_0 = 10 \cdot N$, where N is the number of players sharing the resource.

20 S1.1 Myopic Nash Equilibrium

We can express the Nash Equilibrium solution under myopic conditions, that is the optimal value of Q_i that allows farmer i to maximize their profits knowing that all the other farmers also act maximize theirs. These are myopic conditions in the sense that we assume that all players maximize their profit for period t without regards to the ensuing effect on profits in future periods. We can express these considerations with a system of N first order conditions:

$$25 \quad \frac{\partial U_{it}}{\partial Q_{it}} = 0 \forall i \in N$$

Because all players face identical conditions, their optimized pumping value will be identical. This allows us to express the total amount pumped by the community at period t for player i (which decision we are modelling) and all the other players j , which actions are affecting the costs incurred by player i :

$$Q_t = Q_{it} + (N - 1)Q_{jt}$$

30 Substituting Q_t allows first order conditions to simplify to a single equation:

$$\frac{\partial U_{it}}{\partial Q_{it}} = \alpha - C_{0,t} - Q_{it} - \frac{N - 1}{2} Q_{jt} + \frac{1}{2} = 0$$

Now consider that $Q_{it} = Q_{jt}$ because all players face identical conditions. Plugging that into the first order condition equation and solving, we get Nash Equilibrium pumping rates:

$$Q_{it}^{(NE)} = \frac{\alpha - C_{0,t} + 1/2}{1 + \frac{N-1}{2}}$$

- 35 Plugging in default parameters for *Thirsty Earth* v0 ($\alpha = 50$, $R = 80$, $S_0 = 100$, $k = 1.25$, $N = 10$) yields a myopic Nash Equilibrium pumping rate for period 1 of $Q_{it}^{(NE)} = 9$ units per player and groundwater stocks are exhausted after period 1 in the sense that initial pumping costs on period 2 ($C_{0,t} = 90$) are higher than unit revenues ($\alpha = 50$).

S1.2 Myopic First Best solution

- 40 Alternatively, players can pump water in such a way that total profits aggregated across players are maximized on period t , again not accounting for any effects on profits in subsequent periods. We refer to this solution as a myopic First Best since it maximizes the utility drawn from using the resource (which can be interpreted as water use efficiency) during period t . This amounts to determining total community pumping volume Q_t that maximizes total profits $U_t = \sum_{i \in [1, N]} U_{it}$, so solving first order conditions:

$$\frac{\partial U_t}{\partial Q_t} = \frac{\partial}{\partial Q_t} \left(\alpha Q_t - Q_t \left(C_{0,t} + \frac{Q_t - 1}{2} \right) \right) = 0$$

- 45 We solve this expression and divide by the number of players to obtain individual myopic first best pumping volumes for period t :

$$Q_{it}^{(FB)} = \frac{\alpha - C_{0,t} + 1/2}{N}$$

- Plugging in default parameters for *Thirsty Earth* v0 ($\alpha = 50$, $R = 80$, $S_0 = 100$, $k = 1.25$, $N = 10$) yields a myopic First Best pumping rate for period 1 of $Q_{it}^{(FB)} = 4.95$ units per player. Groundwater stocks are also exhausted after period 1 in the sense
50 that initial pumping costs on period 2 ($C_{0,t} = 49.5$) yield an optimal pumping rate $Q_{it}^{(FB)} = 0$.

S1.3 Steady State Nash Equilibrium and First Best solutions

- The myopic solutions are neither realistic nor desirable, as players might/ought to exercise foresight and restrict current consumption in order to sustain profits in subsequent periods. This behavior can be modeled by adding a constraint to the optimization, namely that groundwater storage should not vary across periods. We can express this by setting to zero the time-derivative
55 of storage S_t :

$$\frac{dS}{dt} = \frac{d}{dt} (R - Q + S/k) = 0$$

This equation expresses the water balance of the system, with R , Q and S/k representing recharge, withdrawals and losses to discharge at each period. Solving the above equation for S yields the storage values that will be maintained across periods under sustainable pumping:

60 $S = (R - Q)k$

We then determine the cost of the first unit of pumped water C_0 that will be achieved at each period maintained groundwater storage to its sustainable level:

$$C_0 = S_0 - S_t = S_0 - (R - Q)k$$

We incorporate this into the profit function:

65 $U_i = \alpha Q_i - Q_i \left(S_0 - (R - Q)k + \frac{Q - 1}{2} \right)$

Note that the subscript t associated with time has been removed. The sustainability constraint on S ensures that the system is maintained at steady state, so pumping and profits will be identical across periods. Crucially, this steady state constraint implies

70 pumping volumes Q_i are set so as to maintain groundwater storage at S indefinitely. This is *not* the optimal strategy for the game played in class, which only consists of 8 periods. This arbitrary number of periods is a practical limitation of the game and we believe that deriving steady state optimal solutions (while making them aware of the distinction) has a higher pedagogical value for students.

We use the steady-state profit function to express the Nash Equilibrium first order conditions:

$$\begin{aligned} \frac{\partial U_i}{\partial Q_i} &= \frac{\partial}{\partial Q_i} \left(\alpha Q_i - Q_i \left(S_0 - (R - Q_i - (N - 1)Q_j)k + \frac{Q_i + (N - 1)Q_j - 1}{2} \right) \right) \\ &= \alpha - S_0 + kR + \frac{1}{2} - (2k + 1)Q_i - (N - 1)(k + \frac{1}{2})Q_j \\ 75 \quad &= 0 \end{aligned}$$

Recall that all players face identical conditions, so $Q = Q_i + (N - 1)Q_j$ in the above equation. We now introduce $Q_i = Q_j$ and solve for the Nash Equilibrium pumping rate under steady state :

$$Q_i^{(NE)} = \frac{\alpha - S_0 + kR + \frac{1}{2}}{(2k + 1) + (N - 1)(k + \frac{1}{2})}$$

80 Using the default parameter values ($\alpha = 50$, $R = 80$, $S_0 = 100$, $k = 1.25$, $N = 10$), we get $Q_i^{(NE)} = 2.62$ and a steady state groundwater storage of $S = 67.125$

Alternatively, steady-state First Best pumping maximizes total community profit conditional on maintaining a constant groundwater storage level. We sum the steady state profit function across the community ($U = \sum_{i \in [1, N]} U_i$) to express first order conditions:

$$\begin{aligned} \frac{\partial U}{\partial Q} &= \frac{\partial}{\partial Q} \left(\alpha Q - Q \left(S_0 - (R - Q)k + \frac{Q - 1}{2} \right) \right) \\ 85 \quad &= \alpha - S_0 + kR + \frac{1}{2} - (2k + 1)Q \\ &= 0. \end{aligned}$$

Solving for Q and dividing by the number of players gives us steady state first best solution:

$$Q_i^{(FB)} = \frac{\alpha - S_0 + kR + \frac{1}{2}}{N(2k + 1)}.$$

90 Using the default parameter values ($\alpha = 50$, $R = 80$, $S_0 = 100$, $k = 1.25$, $N = 10$), we get $Q_i^{(FB)} = 1.44$ and a sustainable groundwater storage of $S = 81.964$.

S2 Thirsty Earth v1: Profit functions and solutions

Similar to *Thirsty Earth* v0, players impersonate farmers seeking to maximize the profit that they can generate over a predetermined number of periods. Unlike *Thirsty Earth* v0, and similar to *Irrigania* (Seibert and Vis, 2012), each farmer must decide during each period whether (and what crop) to plant on each of their 9 fields. There are three irrigation types that yield different profits and costs: rainfed (i.e. non-irrigated), surface water irrigation, and groundwater irrigation. Unlike *Irrigania*, players can also choose to leave a field fallow to earn a fixed income or, in contrast, plant a higher value crop and double the costs and profits arising from the chosen irrigation option. In the subsections below, we define utility (i.e. profit) functions associated with each decision option along with associated optimization concepts (see Figure S1). We also derive constraints on the parameters of utility/profit function that ensure non-trivial decision choices. These constraints are embodied in the default parameters in the online game, which are given in Table S1.

S2.1 Fallow Fields & Rainfed Irrigation

Each period can be either be a *good* year that occurs with a probability P , or a *bad* year that occurs with a probability $1 - P$. The expected profit that a farmer will make from Q_R fields that are rainfed is given by:

$$E[u_R] = (P \cdot \alpha_{R,1} + (1 - P)\alpha_{R,2}) \cdot Q_R$$

where $\alpha_{R,1}$ and $\alpha_{R,2}$ are the net unit profits per field in the event of a good and bad year, respectively. Alternatively, the farmer can choose to leave a field fallow and secure a predetermined wage α_F from outside labor. The profit that the farmer will make from the Q_F fields that are left fallow is:

$$U_F = \alpha_F Q_F$$

Note that the profits U_R and U_F depend solely on the farmer's own decision in the present period. To determine parameters in a way that makes a decision non-trivial, we want the expected profits of rainfed irrigation to be larger than that of an outside wage. We also want profits in a good year to be larger than profits in a bad year. Let $\rho_{RF} > 1$ be the ratio between expected unit returns from rainfed crops and outside wages, and let $\rho_R < 1$ be the ratio between the profit generated by a rainfed field in a bad vs. good year, then free parameters should follow:

$$\alpha_{R,1} = \frac{\alpha_F \cdot \rho_{RF}}{P + \rho_R - \rho_R \cdot P} \quad (1)$$

$$\alpha_{R,2} = \rho_R \cdot \frac{\alpha_F \cdot \rho_{RF}}{P + \rho_R - \rho_R \cdot P} \quad (2)$$

S2.2 Surface Water Irrigation

110 Farmers can obtain water from a shared canal network that they must maintain as a community. Unlike *Irrigania* (Seibert and Vis, 2012), where the revenues from surface water irrigation decrease as the number of participants increases, the farmers make a private revenue that is proportional (by a factor α_S) to the number of fields that they irrigate. However, the *costs* of using the infrastructure is partially public under the presumption that maintenance costs increase with the number of participants. The unit cost per field is dependent on the surface water irrigation consumption of the whole village – the more the infrastructure is being used, the more it costs to maintain it – but heavily weighted by the farmers' own consumption because part of the infrastructure is private (e.g., the canals and equipment used to irrigate their own field) and not maintained by the community. This setup with private revenues and shared costs is an emblematic case of common-pool infrastructure (e.g., Yu et al., 2015). Crucially, surface water irrigation in *Thirsty Earth* v1 has no time dependence, meaning that the profit that each farmer can make on any given period does not depend on their decision on previous periods and does not affect costs in future periods.

120 Profits of farmer i in a given period by applying surface irrigation to $Q_{S,i}$ fields are expressed as:

$$U_{S,i} = \alpha_S Q_{S,i} - Q_{S,i} \left(Q_{S,i} \beta_{S,1} + \sum_{j \neq i} Q_{S,j} \beta_{S,2} \right) \quad (3)$$

where $\beta_{S,1}$ and $\beta_{S,2}$ are multipliers representing the respective effects of the farmer's own consumption on their unit cost of irrigation, and the effect of the other farmers' consumption $Q_{S,j}$.

S2.2.1 Nash Equilibrium and First Best

We can express the Nash Equilibrium solution, that is the optimal value of Q_i that allows farmer i to maximize their profit knowing that all the other farmers also maximize theirs. This can be obtained by solving the first order condition:

$$\frac{\partial U_{S,i}}{\partial Q_{S,i}} = \alpha_S - 2\beta_{S,1} Q_{S,i} - \sum_{j \neq i} Q_{S,j} \beta_{S,2} = 0$$

Because all farmers are subject to the same economic conditions (α_S , $\beta_{S,1}$ and $\beta_{S,2}$), their Nash Equilibrium will be identical ($Q_{S,i} = Q_{S,j} = Q_{S,NS}$), which allows the first order condition to be expressed as:

$$\alpha_S - 2\beta_{S,1}Q_{S,NS} - (N-1)Q_{S,NS}\beta_{S,2} = 0$$

125 where N is the number of farmers in the village. Solving, we get the number of fields that each farmer should irrigate using surface water under the Nash Equilibrium:

$$Q_{S,NS} = \frac{\alpha_S}{2\beta_{S,1} + (N-1)\beta_{S,2}} \quad (4)$$

Alternatively, farmers might decide to maximize community level profits $U_S = \sum_i^N U_{S,i}$, a solution we refer to as *First Best*. The number of fields $Q_{S,i}$ that farmer i would need to irrigate could then be determined by summing all profit functions across
130 all N farmers of the village:

$$U_S = \sum_i^N \left(\alpha_S Q_{S,i} - Q_{S,i} \left(Q_{S,i}\beta_{S,1} + \sum_{j \neq i} Q_{S,j}\beta_{S,2} \right) \right)$$

Then, setting the derivative of that expression with respect to $Q_{S,i}$ equal to zero:

$$\frac{\partial U_S}{\partial Q_{S,i}} = N \left(\alpha_S - 2\beta_{S,1}Q_{S,i} - 2 \sum_{j \neq i} Q_{S,j}\beta_{S,2} \right) = 0$$

Again, because all farmers face identical economic conditions, their First Best decision will be identical so that ($Q_{S,i} = Q_{S,i} = Q_{S,FS}$), which allows the first order conditions to be expressed as:

$$\frac{\partial U_S}{\partial Q_{S,i}} = N (\alpha_S - 2\beta_{S,1}Q_{S,FS} - 2(N-1)Q_{S,FS}\beta_{S,2}) = 0$$

so:

$$Q_{S,FS} = \frac{\alpha_S}{2\beta_{S,1} + 2(N-1)\beta_{S,2}} \quad (5)$$

It follows from the above that $Q_{S,FS} < Q_{S,NS}$ and $U_{S,i}(Q_{S,FS}) > U_{S,i}(Q_{S,NS})$ meaning that, paradoxically, farmers can make more profit by 'collaborating', that is to say, by consuming less then they might have if they sought to maximize their
135 own profit.

S2.2.2 Parametrization

To make the decision non-trivial, we want to set the parameters of the profit function in such a way that a marginal field irrigated with surface water is more profitable than its rainfed alternative, so

$$\rho_{RS} = \frac{\alpha_F \cdot \rho_{RF}}{\alpha_S - \beta_{S,1} - (N-1)\beta_{S,2}} < 1$$

Note that $\alpha_F \cdot \rho_{RF} = E[U_R]$ from the previous section, and the denominator is obtained by plugging in $Q_{S,i} = Q_{S,j} = 1$ in the profit equation. By combining the expressions for Q_{FS} and Q_{NS} , we can also derive the ratios:

$$R_{S,1} = \frac{\beta_{S,1}}{\alpha_S} = \frac{1}{Q_{S,NS}} - \frac{1}{2Q_{S,FS}} > 0 \quad (6)$$

$$140 \quad R_{S,2} = \frac{\beta_{S,2}}{\beta_{S,1}} = \frac{2(Q_{S,NS} - Q_{FS})}{(N-1)(2Q_{S,FS} - Q_{S,NS})} \in [0, 1] \quad (7)$$

The three above expressions can finally be combined to express conditions on the economic parameters for surface water irrigation:

$$\alpha_S = \frac{\alpha_F \cdot \rho_{RF}}{\rho_{RS}} \frac{1}{1 - R_{S,1} - R_{S,1}R_{S,2}(N-1)} \quad (8)$$

$$\beta_{S,1} = \alpha_S \cdot R_{S,1} \quad (9)$$

$$145 \quad \beta_{S,2} = \alpha_S \cdot R_{S,1} \cdot R_{S,2} \quad (10)$$

S2.3 Groundwater Irrigation

Farmers can also obtain irrigation water from their own private wells. As before, farmers make a private profit, but incur costs that are partially public. Although there is no communal maintenance costs like for the surface irrigation case, farmers' pumping decisions do affect the water table depth – and therefore the pumping costs – of others through groundwater connectivity (see, e.g., Mullen et al., 2022). The key difference with the surface water irrigation case, however, is that the system now has a time dependence. Groundwater depth and pumping costs are affected by decisions of all the village's farmers from all the previous game periods. These conditions are comparable to the *River Basin Game* (Hoekstra, 2012), with the differences that (i) there is only one village per game so there is no upstream-downstream relationship, but (ii) players have a larger influence on their own costs compared to the other players' costs. The latter more realistically simulate the cone of depression formed by groundwater under active pumping fields (see, e.g., Müller et al., 2017). Following Mullen et al. (2022), the profit that farmer i makes by allocating $Q_{G,i}$ fields to groundwater irrigation on period t is expressed as:

$$U_{G,i,t} = \alpha_G Q_{G,i} - \beta_G Q_{G,i} \left(d_{i,t} + Q_{G,i} \Psi_1 + \sum_{j \neq i} Q_{G,j} \Psi_2 \right) \quad (11)$$

$$= Q_{G,i} (\alpha_G - \beta_G d_{i,t}) - Q_{G,i}^2 \beta_{G,1} - Q_{G,i} \beta_{G,2} \sum_{i \neq j} Q_{G,j} \quad (12)$$

The second expression admits $\beta_{G,1} = \beta_G \Psi_1$ and $\beta_{G,2} = \beta_G \Psi_2$ and is rearranged to have a similar structure to the profits derived for surface water irrigation. The terms within the parentheses in the first expression represent the time-averaged groundwater depth for farmer i in period t , with $d_{i,t}$ being depth of the water table at the beginning of the period and Ψ_1 and Ψ_2 groundwater responses affected by the storage and conductivity characteristics of the aquifer (see Mullen et al., 2022). We assume that Ψ_2 represents the average unit effect of another farmer's pumping on farmer i 's average water depth during that same period.

165 S2.3.1 Myopic Nash Equilibrium

We seek to determine the *myopic Nash Equilibrium*, which is the number of fields a farmer might irrigate with groundwater to maximize their utility on period t , knowing that all the other farmers do the same thing. This Nash Equilibrium is qualified as *myopic* because it assumes that all farmers maximize their profit at period t without accounting for their ability to generate profit in subsequent periods. Similar to the surface water irrigation case, the Nash Equilibrium solution for period t can be obtained by solving first order conditions:

$$\frac{\partial U_{G,i,t}}{\partial Q_{G,i}} = \alpha_G - \beta_G d_t - 2\beta_{G,1} Q_{G,i} - \sum_{j \neq i} Q_{G,j} \beta_{G,2} = 0$$

Since all players face identical economic conditions, we can express Nash Equilibrium allocations $Q_{G,NG0} = Q_{G,i} = Q_{G,j}$ for period 1 (where $d_0 = 0$) as:

$$Q_{G,NG0} = \frac{\alpha_G}{2\beta_{G,1} + (N-1)\beta_{G,2}} \quad (13)$$

$$175 \quad = \frac{\alpha_G}{\beta_{G,1}} \cdot \frac{1}{2 + (N-1)\frac{\beta_{G,2}}{\beta_{G,1}}} \quad (14)$$

A comparable expression can be derived for myopic first-best pumping using a similar approach as in Section S1.

S2.3.2 Steady state Nash Equilibrium and First Best solutions

Alternatively, farmers might optimize their profits with foresight, that is by adding a constraint on their consumption to make sure that they are able to maintain the optimized profits indefinitely. Unlike surface water irrigation, groundwater irrigation has a time dependence, where depth to groundwater is determined by groundwater depth at the previous period and the pumping history of all the farmers in the village. Groundwater is also affected by recharge, which on any given year takes either of two values (r_1 or r_2) depending on whether this is a *good* or *bad* year (note that in the expression below, $r_2 = r_1/\rho_{Re}$ with ρ_{Re} the ratio between recharge in good and bad years). Similar to profits from rainfed crops, we can use the probability P of t being a good year to get the expectation of recharge:

$$E[r] = P \cdot r_1 + (1 - P) \cdot r_1/\rho_{Re}$$

We can now express expected changes in water depths using water balance considerations:

$$\frac{\partial d_i}{\partial t} = \Psi_1 Q_{G,i} + \Psi_2 \sum_{i \neq j} Q_{G,j} - E[r] + \frac{d_m - d_i}{k}$$

The last term of the expression represents groundwater loss to base flow production, with d_m the depth to the bottom of the aquifer and k a recession constant. A farmer can choose to exercise foresight and account for this time dependence in their water allocation decision. Specifically, a *sustainable* level of pumping satisfies $\frac{\partial d_i}{\partial t} = 0$, so:

$$d = k \left(\Psi_1 Q_i + \Psi_2 \sum_{i \neq j} Q_j - E[r] \right) + d_m \quad (15)$$

Similar to *Thirsty Earth* v0, this represents the groundwater depth that should be maintained in an indefinitely long steady state, but may not represent the optimal strategy for a game with a finite number of periods.

Steady state Nash Equilibrium allocations can now be determined by plugging the sustainability condition on d into the utility function

$$U_{G,i,t} = Q_{G,i} \alpha_G - Q_{G,i} \beta_G \left(k \left(\Psi_1 Q_{G,i} + \Psi_2 \sum_{i \neq j} Q_{G,j} - E[r] \right) + d_m \right) - Q_{G,i}^2 \beta_{G,1} - Q_{G,i} \beta_{G,2} \sum_{i \neq j} Q_{G,j} \quad (16)$$

$$= Q_i \alpha_G - Q_i^2 \beta_{G,1} k - Q_i \beta_{G,2} k \sum_{i \neq j} Q_j + Q_i \beta_G (k E[r] - d_m) - Q_{G,i}^2 \beta_{G,1} - Q_{G,i} \beta_{G,2} \sum_{i \neq j} Q_{G,j} \quad (17)$$

Then, setting the derivative of that expression with respect to $Q_{G,i}$ equal to zero:

$$\frac{\partial U_{G,i}}{\partial Q_{G,i}} = \alpha_G - 2Q_{G,i} \beta_{G,1} k - \beta_{G,2} k \sum_{i \neq j} Q_{G,j} + \beta_G (k E[r] - d_m) - 2Q_{G,i} \beta_{G,1} - \beta_{G,2} \sum_{i \neq j} Q_{G,j} = 0$$

Again, assuming that $Q_{G,i} = Q_{G,j} = Q_{G,NG}$, we can express Sustainable Nash Equilibrium allocations Q_{NG} as:

$$Q_{G,NG} = \frac{\alpha_G + \beta_G (k E[r] - d_m)}{2\beta_{G,1}(k+1) + \beta_{G,2}(N-1)(k+1)} \quad (18)$$

$$= \frac{\frac{\alpha_G}{\beta_{G,1}} + \frac{\beta_G}{\beta_{G,1}} (k E[r] - d_m)}{2(k+1) + \frac{\beta_{G,2}}{\beta_{G,1}} (N-1)(k+1)} \quad (19)$$

Similarly, the first order conditions for sustainable first best allocation is:

$$\frac{\partial U_{G,i}}{\partial Q_{G,i}} = \alpha_G - 2Q_{G,i}\beta_{G,1}k - 2\beta_{G,2}k \sum_{i \neq j} Q_{G,j} + \beta_G(kE[r] - d_m) - 2Q_{G,i}\beta_{G,1} - 2\beta_{G,2} \sum_{i \neq j} Q_{G,j} = 0$$

205 with solution

$$Q_{G,FG} = \frac{\alpha_G + \beta_G(kE[r] - d_m)}{2\beta_{G,1}(k+1) + 2\beta_{G,2}(N-1)(k+1)} \quad (20)$$

$$= \frac{\frac{\alpha_G}{\beta_{G,1}} + \frac{\beta_G}{\beta_{G,1}}(kE[r] - d_m)}{2(k+1) + 2\frac{\beta_{G,2}}{\beta_{G,1}}(N-1)(k+1)} \quad (21)$$

S2.3.3 Parametrization

210 We seek to derive the parameters of the utility function, namely α_G , $\beta_{G,1}$, $\beta_{G,2}$, β_G , d_m , r_1 and r_2 . As before, a non-trivial decision can be ensured by making the marginal groundwater-irrigated field more profitable than both the surface water (which does not have memory effect) and rainfed (which does not have common-pool effects) alternatives. This can be done by deriving the ratio ρ_{RG} between the marginal utilities of the first rainfed vs. groundwater-irrigated field

$$\rho_{RG} = \frac{\alpha_F \cdot \rho_{RF}}{\alpha_G - \beta_G d_m - \beta_{G,1} - (N-1)\beta_{G,2}} < \rho_{RS} < 1$$

215 Again, note that $\alpha_F \cdot \rho_{RF} = E[U_R]$ represents the expected utility from a rainfed field, and the denominator in the above expression can be obtained by setting $Q_{G,i} = Q_{G,j} = 1$ and $d_{i,t} = d_m$ in the groundwater profit function. In addition, combining the expressions for $Q_{G,NG0}$, $Q_{G,NG}$, and $Q_{G,FG}$ gives the ratios:

$$R_{G,1} = \frac{\beta_{G,1}}{\alpha_G} = \frac{2 - \frac{Q_{G,NG}}{Q_{G,FG}}}{2Q_{G,NG0}} \quad (22)$$

$$R_{G,2} = \frac{\beta_{G,2}}{\beta_{G,1}} = \frac{-2(Q_{G,FG} - Q_{G,NG})}{(N-1)(2Q_{G,FG} - Q_{G,NG})} \in [0, 1] \quad (23)$$

$$R_{G,3} = \frac{\beta_G}{\beta_{G,1}}(kE[r] - d_m) = \frac{2Q_{G,FG}(Q_{G,NG} + kQ_{G,NG} - Q_{G,NG0})}{2Q_{G,FG} - Q_{G,NG}} \quad (24)$$

220 The above expressions can finally be combined to express three of the economic parameters:

$$\alpha_G = \frac{\alpha_F \cdot \rho_{RF}}{\rho_{RG}} \frac{1}{1 - R_{G,1}(R_{G,3} - kQ_{G,NG} - (N-1)R_{G,2}(kQ_{G,NG} + 1) - 1)} \quad (25)$$

$$\beta_{G,1} = \alpha_G \cdot R_{G,1} \quad (26)$$

$$\beta_{G,2} = \alpha_G \cdot R_{G,1} \cdot R_{G,2} \quad (27)$$

The above considerations leave the following free parameters, with default values in Table S1:

- 225
- $Q_{G,NG0}$, $Q_{G,NG}$, $Q_{G,FG}$: the number of fields allocated to groundwater irrigation under Myopic Nash Equilibrium pumping, and under sustainable Nash Equilibrium and First Best pumping solutions, respectively.
 - ρ_{RG} : the ratio of the initial marginal profits for a field allocated to rainfed cropping vs groundwater irrigation.
 - $\rho_{Re} < 1$: The ratio between the groundwater recharge in a bad vs. good year, to be used to determine r_1 and r_2 given $E[r]$ and P .

- 230 – k : the recession constant that is the inverse of the proportion of the stored groundwater lost to discharge each period (e.g., $k=1.25$ means that 20% of the stored groundwater discharges every year).
- $E[r]$: the average annual recharge in terms of "field equivalent". For example, $E[r] = 10$ means that the water equivalent to the consumption of 10 fields is recharged every year for the whole village (i.e. 2 fields per farmer if the village has 5 farmers).
- 235 – λ : the ratio between the maximum possible losses (i.e. losses to baseflow if the groundwater reservoir is completely full) and the average groundwater recharge. It describes the relative water level at natural steady state (i.e. if $\lambda=1$, the reservoir is generally completely full at steady state; if $\lambda=0.5$, the reservoir is half full, etc). This parameter is used to determine d_m given $E[r]$ and k .

S3 Thirsty Earth v1: Software Implementation Details

- 240 *Thirsty Earth* v1 is implemented as a web application written in React JavaScript using the Boardgame.io engine. The production build of *Thirsty Earth* v1 is served on a machine owned and maintained by the Center for Research Computing at the University of Notre Dame. The code base is served through a cluster of four Docker containers: a client container which houses the game engine code, an Nginx container which serves the application at a public web access point, a Postgres container which stores data about gameplay sessions, and a Plumber container which houses the R model code that runs in the background to
- 245 determine point distribution to players at the end of each round, allowing it to be accessed as an API endpoint.

S3.1 Code Architecture

- The majority of the *Thirsty Earth* v1 code was written following the Boardgame.io engine specifications. Boardgame.io is a JavaScript library that provides tools for creating turn-based digital board games which can be played in a web browser. The game engine is made up of configuration and web component files which interact with each other through Boardgame.io
- 250 classes. The viewable pages are React components which reference and interact with the functions in a `Game.js` configuration file. This file includes utilities for setting up a game and moving through various 'phases' of play where players can perform an allowed set of 'moves'. Routes to the various page components are registered using the React Router functions in the `App.js` file. The redirect flow of the web components takes an 'instructor' user through a game creation page to a moderator view where they can share a Game ID generated by Boardgame.io with players (students) so they can join the newly created game.
- 255 The view for moderators and players is segmented.

S3.2 Moderator Features

- There are a few actions available to the moderator during the setup phase which include: filling empty seats with BOT players, running the R model to generate public information for the game based on information entered at game creation, adding information bits to be displayed to players in a village at the request of players in that village, and starting the game. At the
- 260 end of each round of play, the R model runs in the background through a button input from the moderator which sends a list of POST requests (one per village) to the Plumber endpoint. These requests are sent as a promise which expects a response back before proceeding to the next phase where players are able to view their previous year summary. Play proceeds until the final year is reached, at which point a summary of the player's profits is shown to each player. A log of the game may be downloaded by the moderator in CSV format. This CSV is generated by a script which retrieves the game information from the Postgres
- 265 database and formats it.

S3.3 Additional Features

The *Thirsty Earth* v1 platform includes additional features not directly related to gameplay. On the left of the player board is a chat function for both communication between players in the same village and globally to all players. The moderator can view all these chats. On the right, each player also sees a history of their actions from previous rounds. Before creating a new game,

270 the game creator may generate a graph which displays the optimal profits for that game configuration. Lastly, the moderator can rewind the game state to a previous year, wiping the current year’s player data. This is useful in instances where a trial round may be necessary for new players to understand the gameplay loop.

S3.4 Limitations and Challenges

275 While *Thirsty Earth* v1 plays relatively smoothly and has been used in real classroom settings, there are some caveats and limitations of this platform. For one, the game engine used, Boardgame.io, relies on active development from a dedicated volunteer team, which could affect the future stability of the platform. The engine itself also has limitations that became apparent during development. Boardgame.io does not allow asynchronous calls, which means that the model cannot run truly 'in the background'. It requires interaction from the client, which makes running the game without a moderator or with a timer difficult, since the function run cannot be automated. Boardgame.io is also a client-side application so the game state is tracked through session data. This means that if a player is booted from the game, as in the case of a browser crash or the like, they cannot rejoin as the same player since the player identification information for their game is stored in cookies, which may not persist between browser sessions. To try and remedy this, there are extra player seats included in each game which can be put in place of a disconnected player. This does not completely solve the problem, however, because the player’s data from previous years will exist on their disconnected session and data from those two players has to be patched together to get an accurate reading of the data. Lastly, there was a degradation in user interface responsiveness correlated with the number of players in the game. The drop off in performance becomes noticeable at >50 concurrent players and most likely occurs because of an increased load on server resources from the background API calls to run the R model. To mitigate confusion and limit excess strains on performance, there is a hard cap on game participants at 100, meaning the total number players (minus the moderator) must be less than or equal to 99.

290 **S4 Synthesis of student feedback**

The report below was generated by ChatGPT-4o based on an analysis of 69 anonymized forum posts of approximately 500 words each submitted by 54 students enrolled in two engineering classes at the University of Notre Dame between 2021 and 2023. Posts were submitted via the course’s learning management system (Canvas) following two gameplay sessions of the Thirsty Earth irrigation simulation. The first set of posts was written after the initial game session (without institutional rules), and the second followed a round played after student-led institutional design workshops. The posts represent reflections from distinct subsets of students before and after the introduction of governance mechanisms. Prior to analysis, all identifying information was removed and student names were replaced with initials to ensure anonymity. ChatGPT was then prompted as follows:

300 Here are student forum posts. Can you distill and synthesize the content? Write a report with the main themes and relevant verbatim quotes.

Strategy and Performance

- **Conservative Approaches:** Many students recognized the need for cautious water use to avoid financial losses.
 - “I knew not to take anything so as to not lose money... I decided to play a little more conservatively.” – H.E.
 - “We were dependent on [other community members]... I ended up having to be more conservative to try and make a profit.” – J.M.

This behavior was not only strategic but also reflective of growing awareness of the shared and interdependent nature of common-pool resources.

- **Exploitation for Gain:** Several students described using deception or defection to maximize their own profits.
 - “I played the role of the bad guy and took out more water than I led on... it worked pretty well.” – O.R.
 - “I decided to see what would happen if I lied to my neighbors... clearly, it benefited me.” – E.J.

These tactics suggests that students have experienced incentives to freeride that are typical to common-pool resources.

- **Adaptive Learning:** Across both rounds (before and after institutional design), students demonstrated a clear evolution in strategy.
 - “We started out taking a lot and lost money... then adjusted and made small, consistent profits.” – A.L.
 - “We agreed on an amount and adjusted it when the price got too high... it helped us all profit.” – L.M.

Groups noticed that they performed better after designing institutions.

Cooperation and Trust

- **Group Agreements and Norms:** Most groups established collective extraction limits during institution design.
 - “We went off of an honor code system... and it worked because no one felt the need to break it.” – A.L.
 - “Our sanction was that if the rule was broken, all profits would be pooled and redistributed... this discouraged cheating.” – E.J.
- **Formal Sanctions and Monitoring:** Some groups used structured enforcement mechanisms (thus demonstrating mastery of Ostrom’s design principles).
 - “No one wanted to deal with the consequences... the average was always the agreed amount.” – D.T.
 - “We had a fine of \$5 per violation, but looking back, it should have been higher... no one broke the rule, thankfully.” – H.E.
- **Trust Challenges:** Despite cooperative intentions, students expressed skepticism about compliance.
 - “There’s no way to know if someone lied... I assumed someone did, so I took a bit more too.” – O.R.
 - “We all followed the rules, not because of the fine, but because we felt responsible for each other.” – L.A.

These experiences underscore the difficulty of relying solely on informal social norms.

Uncertainty and Risk

- **Rainfall Variability:** Students frequently cited the unpredictability of rainfall as a key challenge.
 - “The uncertainty of the rain was quite frustrating... I had no idea how much water would be available each round.” – H.E.
 - “We couldn’t anticipate how much it would rain... or how much the upstream group would take.” – C.W.
- **Behavioral Uncertainty:** Students also struggled with anticipating others’ behavior.
 - “We bought information each round, but the cost of doing so actually hurt us more than it helped.” – Y.Q.

Connection to Course Concepts

- **Groundwater Depletion and Equity:** Students linked game experiences to real-world problems.
 - “If one person is taking more than their fair share, the whole system could collapse.” – O.R.
 - “Taking too much water one year leads to higher prices the next... that helped us understand depletion.” – L.M.
- **Institutional Design and Enforcement:** Students explored rule-making challenges.
 - “We thought the rules were fair... but it was hard to enforce them without knowing who broke them.” – E.J.
 - “We all agreed on a maximum, but the cost of buying information drained our profits.” – C.C.
- **Ostrom’s Framework and Collective Governance:** Several students referenced Elinor Ostrom explicitly.
 - “Ostrom showed that people can work together—but only if they’re willing to put group needs ahead of their own.” – A.L.
 - “Trust and transparency are key... but in reality, people are often too selfish to follow the rules voluntarily.” – E.J.

Key Takeaways

1. **Cooperation works—but only with trust or enforcement.** Social cohesion or institutional sanctions are necessary to deter overuse.
2. **Rules must be monitored to be meaningful.** Information asymmetries undermine even well-designed regulations.
3. **Rainfall and behavioral uncertainty are key risks.** Players must make decisions with incomplete knowledge of both resource availability and others’ actions.
4. **Real-world water governance is complex.** The simulation illustrates that even with good intentions, sustainability is hard to achieve without alignment of incentives.
5. **Short-term self-interest often prevails.** Without strong norms or sanctions, individuals tend to prioritize their own gain, even at collective cost.

Table S1. *Thirsty Earth v1* Game Parameters.

Parameter	Default	Description
General Parameters		
Number of Villages	–	Number of villages to create.
Players per Village	–	Number of player seats to create for each village.
Number of Years	–	Number of rounds in the game.
Game Nickname	–	Game label for data export.
Climate Parameters		
P	0.5	Probability of a good rain year.
L_d	1.25 years	Average number of consecutive bad rain years.
dP	0	Annual increase in P.
dL_d	0 years	Annual increase in L_d .
Economic Parameters		
ρ_{RF}	1.2	Ratio between expected unit returns from rainfed crops and outside wages (fallow profits).
ρ_{RS}	0.1	Ratio of utilities for rainfed crops and surface water crops.
ρ_{RG}	0.06	Ratio of utilities for rainfed crops and groundwater crops.
ρ_R	0.15	Ratio of rainfed profits in a bad vs. good rain year.
ρ_{Re}	0.8	Ratio of groundwater recharge for a good vs. bad year.
aCr	2	High value crop multiplier.
aPen	0.5	Percentage of the profit if all fields were fallow.
Q_{NS}	4 fields	Nash equilibrium (selfish) surface water optimum.
Q_{FS}	3 fields	First best (community-oriented) surface water optimum.
Q_{NG0}	5 fields	Year one (short term) Nash equilibrium groundwater optimum.
Q_{NG}	3 fields	Sustainable (long term) Nash equilibrium groundwater optimum.
Q_{FG}	2 fields	Sustainable first best groundwater optimum.
α_F	\$1.00	Profit per field left fallow.
EPR	3 m	Expected groundwater recharge.
k	1.75	Recession constant per period representing groundwater lost to environmental flows.
lambda	0.9	The ratio between the maximum possible environmental outflow and the average groundwater recharge.
Calculated Parameters		
α_{R1}	\$2.09	Rain unit benefit in a good year.
α_{R2}	\$0.31	Rain unit benefit in a bad year.
α_S	\$14.4	Surface water unit benefit.
β_{S1}	\$1.2	Surface water cost multiplier for your own consumption.
β_{S2}	\$0.24	Surface water cost multiplier for other villagers' consumption.

Continued on next page

Table S1: *Thirsty Earth* Game Parameters (continued)

Parameter	Default	Description
α_G	\$28.07	Groundwater unit benefit.
β_{G1}	\$1.40	Groundwater cost multiplier for your own consumption.
β_{G2}	\$0.56	Groundwater cost multiplier for other villagers' consumption.
β_G	\$34.75	Groundwater cost multiplier for depth to the water table.

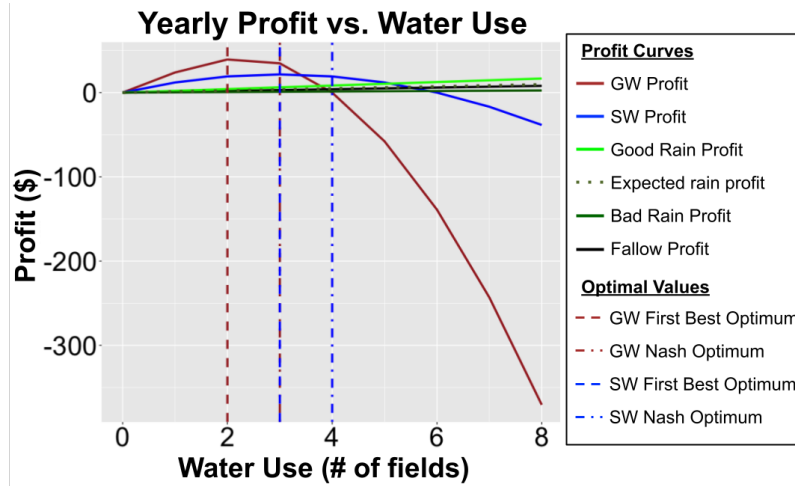


Figure S1. Individual annual profits for each irrigation source assuming all other players choose the same quantities. The vertical lines represent the Nash Equilibrium (parabola summits) and First Best options for groundwater (GW) and surface water (SW). The expected value of rainfed profits is also plotted alongside the rainfed profits for a good or bad rain year.

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