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# Technical Note: Revisiting the general calibration of cosmic-ray neutron sensors to estimate soil water content

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## S1. Partial derivatives

In this section, we provide the partial derivatives of $\theta^G$ (Eq. 2 in the technical note) and $\theta^L$ (Eq. 10 in the technical note). For the sake of comprehensibility, we will reproduce the underlying equations for $\theta^G$ and $\theta^L$ at the beginning of the following sub-sections.

### S1.1 Partial derivatives of $\theta^G$

The following is the equation for the general calibration function:

$$\theta^G(N) = \Big(\frac{a_0}{f_p \cdot f_h \cdot f_{\text{in}} \cdot f_b \cdot f_s \cdot \frac{N}{N_0} - a_1} - a_2 - \theta_g^{\text{OM}} - \theta_g^{\text{LW}}\Big) \cdot \frac{\rho_b}{\rho_w} \tag{S1}$$

Assuming $f_p = f_h = f_{\text{in}} = 1$ and including Eqs. 6-9 from the technical note, the partial derivatives required for the error propagation result to the following:

$$\frac{\partial\theta}{\partial N} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot f_s \cdot N_0}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{S2}$$

$$\frac{\partial\theta}{\partial N_0} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot f_s \cdot N}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{S3}$$

$$\frac{\partial\theta}{\partial f_s} = \frac{0.009^{-1} \cdot a_0 \cdot \rho_b \cdot (\text{AGB} - 0.009^{-1}) \cdot N \cdot N_0}{\rho_w \cdot (a_1 \cdot (\text{AGB} - 0.009^{-1}) \cdot N_0 + 0.009^{-1} \cdot f_s \cdot N)^2} \tag{S4}$$

$$\frac{\partial\theta}{\partial \text{AGB}} = \frac{0.009 \cdot a_0 \cdot \rho_b \cdot f_s \cdot N \cdot N_0}{(\rho_w \cdot (a_1 \cdot N_0 \cdot (0.009 \cdot \text{AGB} - 1) + f_s \cdot N)^2)} \tag{S5}$$

$$\frac{\partial\theta}{\partial \rho_b} = \Big(\frac{a_0}{\frac{f_s \cdot N}{N_0 \cdot (1 - \text{AGB} \cdot 0.009)} - a_1} - a_2 - s \cdot 0.556 - \theta_g^{LW}\Big) \cdot \frac{1}{\varrho_w} \tag{S6}$$

$$\frac{\partial\theta}{\partial OM} = -0.556 \cdot \frac{\rho_b}{\rho_w} \tag{S7}$$

$$\frac{\partial\theta}{\partial \theta_g^{LW}} = -\frac{\rho_b}{\rho_w} \tag{S8}$$

## S1.2 Partial derivatives of $\theta^L$

The following is the equation for the general calibration function:

$$\theta^L(N) = \left( a_0 \cdot \left( \frac{\tau \cdot N}{\tau_{\text{cal}} \cdot N_{\text{cal}}} \cdot \left( \frac{a_0}{\theta_{\text{cal}} \cdot \frac{\rho_w}{\rho_b} + a_2} + a_1 \right) - a_1 \right)^{-1} - a_2 \right) \cdot \frac{\rho_b}{\rho_w} \tag{S9}$$

Assuming $\tau = \tau_{\text{cal}} = 1$, the partial derivatives required for the error propagation result to the following:

$$\frac{\partial \theta}{\partial N} = -\frac{a_0 \cdot \rho_b \cdot N_{\text{cal}} \cdot (a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) \cdot (a_0 \cdot \rho_b + a_2 \cdot a_1 \cdot \rho_b + a_1 \cdot \theta_{\text{cal}} \cdot \rho_w)}{\rho_w \cdot (a_2 \cdot a_1 \cdot \rho_b \cdot (N_{\text{cal}} - N) - a_0 \cdot \rho_b \cdot N + a_1 \cdot \theta_{\text{cal}} \cdot \rho_w \cdot (N_{\text{cal}} - N))^2} \tag{S10}$$

$$\frac{\partial \theta}{\partial N_{\text{cal}}} = \frac{a_0 \cdot \rho_b \cdot N \cdot (a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) \cdot (a_0 \cdot \rho_b + a_2 \cdot a_1 \cdot \rho_b + a_1 \cdot \theta_{\text{cal}} \cdot \rho_w)}{\rho_w \cdot (a_1 \cdot (N - N_{\text{cal}}) \cdot (a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) + a_0 \cdot \rho_b \cdot N)^2} \tag{S11}$$

$$\frac{\partial \theta}{\partial \theta_{\text{cal}}} = \frac{a_0^2 \cdot \rho_b^2 \cdot N \cdot N_{\text{cal}}}{(a_1 \cdot (N - N_{\text{cal}}) \cdot (a_2 \cdot \rho_b + \rho_w \cdot \theta_{\text{cal}}) + a_0 \cdot \rho_b \cdot N)^2} \tag{S12}$$

$$\begin{aligned} \frac{\partial \theta}{\partial \rho_b} = & \frac{1}{\rho_w \cdot (a_1 \cdot (N - N_{\text{cal}}) \cdot (a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) + a_0 \cdot N \cdot \rho_b)^2} \cdot ((a_1 \cdot (N - N_{\text{cal}})(a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) + a_0 \cdot N \cdot \rho_b) \\ & \cdot (a_0 \cdot N_{\text{cal}}(a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) - a_2 \cdot (a_1 \cdot (N - N_{\text{cal}}) \cdot (a_2 \cdot \rho_b + \theta_{\text{cal}} \cdot \rho_w) + a_0 \cdot N \cdot \rho_b)) \\ & - a_0^2 \cdot N_{\text{cal}} \cdot N \cdot \theta_{\text{cal}} \cdot \rho_w \cdot \rho_b) \end{aligned} \tag{S13}$$