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Technical note: How physically based is hydrograph separation by recursive digital filtering?

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Abstract. Recursive digital filtering of hydrographs is a widely used method to identify streamflow components, which react to precipitation with varying degrees of attenuation and delay. In this context, a distinction is often made between physically based and non-physically based algorithms. A well-known example of a physically based algorithm is that of Furey and Gupta (2001). In this paper, it is contrasted with the widely used algorithm of Eckhardt (2005). This algorithm is often considered merely a non-physically based low-pass filter. However, the comparison shows that both algorithms largely agree. The algorithm of Eckhardt (2005) differs from the algorithm of Furey and Gupta (2001) only in the time delay assumed between precipitation and the exfiltration of baseflow into surface waters and in the fact that two parameters are combined into one, BFI_{max}. This parameter can thus be interpreted physically, and an approach for its calculation emerges.

1 Introduction

A catchment can be understood as a signal converter. The precipitation is the input signal that is converted into the output signal, streamflow. In the course of this signal conversion, the water takes different paths through the catchment and is subject to different hydrological processes. This results in streamflow components that are attenuated and delayed to varying degrees compared to the input signal, the precipitation. Usually, two components are distinguished: on the one hand, the so-called baseflow as a low-frequency signal component and, on the other hand, higher-frequency signal components that are generated more quickly and are less attenuated in response to precipitation events, the so-called direct runoff. From this idea, it is obvious that low-pass filtering of streamflow hydrographs can be used to identify these components.

This approach has been followed since Lyne and Hollick (1979) introduced the recursive digital low-pass filter to hydrology. The term "digital" refers to the fact that discrete, equidistant-in-time data of the streamflow are used, the processing of which can be easily automated by using a computer. The term "recursive" refers to the fact that the signals of the preceding time steps are included in the calculation of the output signal in the current time step.

Several such recursive digital low-pass filters were subsequently presented. In the following, the filter developed by Eckhardt (2005) is considered in particular. It is now one of the established methods of hydrograph separation – for example, as part of the US Geological Survey Hydrologic Toolbox (Barlow et al., 2022).

The "Eckhardt filter", as it is oftentimes called, is usually counted among the non-physical or "purely empirical" (Healy, 2010, p. 87) methods of hydrograph separation. The apparent lack of a physical basis repeatedly raises doubts about the justification of the recursive digital filtering: "Most hydrograph separations (apart from tracer-based separations) lack a physical basis. [. . .] Therefore, choosing one method or the other introduces an undesirable element of uncertainty and randomness into the analysis and comparison of runoff coefficients" (Blume et al., 2007). "The digital filter methods have no physical meaning" (Kang et al. 2022). However, without a physically meaningful interpretation, it becomes impossible to objectively determine the parameters of the filter algorithms: "parameters used in the RDF [recursive digital filtering] method are often determined arbitrarily, resulting in high uncertainty of the estimated baseflow

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rate." (Zhang et al., 2013), "quantitative results of the filtering change with the value of the parameters. Although the shape of the hydrograph separation can be visually consistent with the conceptualisation of a hydrograph separation, it is basically impossible to draw any conclusion from it." (Pelletier and Andréassian, 2020), "To accurately separate the baseflow from streamflow with the digital filter methods, appropriate filter parameters must be estimated by trial and error, which act as a difficulty or limitation on their use." (Kang et al., 2022).

Is this criticism justified? Does the widespread recursive digital filtering, especially that with Eckhardt's algorithm, really lack a physical, hydrologically plausible explanation, and does the choice of parameter values remain arbitrary?

In order to shed light on the answers to these questions, Eckhardt's filter is compared below with the algorithm of Furey and Gupta (2001). The latter has been developed explicitly from hydrological principles. Its developers therefore – rightly – describe it as physically based and emphasise the difference to the previously mentioned low-pass filters: "Unlike other filters, our filter is not founded on the assumption that base flow and overland flow are the low- and high-frequency components of streamflow, respectively." The analysis shows that there is nevertheless a close relationship between the Eckhardt (2005) and Furey and Gupta (2001) filters and thus provides a clue as to how the parameter $\mathrm{BFI}_{\max}$ of the recursive digital filter of Eckhardt (2005) can be physically interpreted and determined.

2 The two separation methods

2.1 The method of Eckhardt (2005)

The equation of this low-pass filter is

$$b_k = \frac{(1-\mathrm{BFI}_{\max})ab_{k-1} + (1-a)\mathrm{BFI}_{\max}y_k}{1-a\mathrm{BFI}_{\max}}, \tag{1}$$

where b is the baseflow, y is the streamflow, k is the time step number, and a and $\mathrm{BFI}_{\max}$ are parameters whose values must be set before applying the filter. Equation (1) is subject to the condition $b_k \leq y_k$, that is, if, which is mathematically possible, $b_k > y_k$ results, $b_k = y_k$ is set.

Even though the filter of Eckhardt (2005) is contrasted here with the filter of Furey and Gupta (2001), which is explicitly described as physically based, it is nevertheless also itself based on plausible assumptions:

a. The information about the baseflow b_k of the current time step k lies in the baseflow b_{k-1} of the preceding time step $k-1$ and in the total streamflow y_k of the current time step:

$$b_k = Ab_{k-1} + By_k, \tag{2}$$

with parameters A and B that are functions of the filter parameter a and for which $A > 0$ and $B > 0$ are assumed (Eckhardt, 2005; Eq. 8).

b. Baseflow is runoff from a linear reservoir, i.e. it is proportional to the amount of water stored in this reservoir. The filter parameter a corresponds to the so-called recession constant of the reservoir, which can be derived from the streamflow data, as described in Eckhardt (2008).

c. The algorithm of Lyne and Hollick (1979) has been criticised as hydrologically implausible, since it shows a constant streamflow y or baseflow b, respectively, when direct runoff $y-b$ has ceased (Chapman, 1991). Equation (1) does not have this disadvantage: from $y_k - b_k = 0$ or $y_k = b_k$ follows

$$y_k = \frac{(1-\mathrm{BFI}_{\max})ab_{k-1} + (1-a)\mathrm{BFI}_{\max}y_k}{1-a\mathrm{BFI}_{\max}}.$$

This equation can be simplified to $y_k = ab_{k-1}$ or, since in this situation the streamflow consists entirely of baseflow,

$$b_k = ab_{k-1}. \tag{3}$$

This is exactly the equation that describes the exponential decrease in runoff from a linear reservoir.

d. The second filter parameter $\mathrm{BFI}_{\max}$ is the maximum value of the baseflow index (the long-term ratio of baseflow to total streamflow) that can be calculated with the filter algorithm. This maximum value is less than 1. This too is plausible. A catchment with a baseflow index of 1, i.e. a catchment without direct runoff, would have to have a soil with an extremely high infiltration and storage capacity and/or would have to be flat. In such an area, there would be no watercourse at all whose baseflow index could be determined.

The calculation with Eckhardt's algorithm requires streamflow data and the values of two parameters, with the streamflow data allowing one of the two parameters, the recession constant a, to be determined. How uncertainties in the two filter parameters affect the resulting baseflow index can be calculated as Eckhardt (2012) has shown.

2.2 The method of Furey and Gupta (2001)

Furey and Gupta formulated their filter algorithm as

$$\overline{Q}_{\mathrm{B},j} = (1-\gamma)\overline{Q}_{\mathrm{B},j-1} + \gamma\frac{c_3}{c_1}\left(\overline{Y}_{\mathrm{B},j-d-1} - \overline{Q}_{\mathrm{B},j-d-1}\right) \tag{4}$$

(Furey and Gupta, 2001; Eq. 22). $\overline{Q}_{\mathrm{B},j}$ is the baseflow at time step j, $1-\gamma$ is the recession constant, c_1 and c_3 are the proportions of precipitation that become overland flow and groundwater recharge, $\overline{Y}_{\mathrm{B},j-d-1}$ is the streamflow at time step $j-d-1$, and d is the delay between precipitation and groundwater recharge. Using the same symbolic designation for time step number, recession constant, baseflow, and streamflow as in Eq. (1), Eq. (4) can also be written as

$$b_k = ab_{k-1} + (1-a)\frac{c_3}{c_1}(y_{k-d-1} - b_{k-d-1}). \tag{5}$$

The calculation of the baseflow according to Furey and Gupta (2001) requires streamflow and precipitation data and the values of four parameters: a, c_1, c_3, and d. Precipitation is needed for the derivation of the values of c_1 and c_3. How d can be estimated remains open.

2.3 The relation between the two algorithms

In deriving their filter equation, Furey and Gupta (2001) assume that the baseflow in the current time step is a function of baseflow and groundwater recharge one time step in the past (their Eq. 10). Further, they assume that the groundwater recharge is delayed by d time steps compared to precipitation (their Eq. 11). In their model of the emergence of baseflow, the number of time steps between precipitation and baseflow is $d+1$: d time steps between precipitation and groundwater recharge $+1$ time step between groundwater recharge and baseflow, hence the index $j-d-1$ in Eq. (4) or $k-d-1$ in Eq. (5).

If instead it is assumed that baseflow occurs in the same time step as groundwater recharge and that groundwater recharge is not delayed compared to precipitation (in other words, if it is assumed that the delay between precipitation and baseflow is smaller than one time step), then $d+1=0$ and thus $k-d-1 = k-(d+1) = k$. Equation (5) is then

$$b_k = ab_{k-1} + (1-a)\frac{c_3}{c_1}(y_k - b_k). \tag{6}$$

This equation can be transformed to

$$b_k = \frac{a}{1+(1-a)\frac{c_3}{c_1}} b_{k-1} + \frac{(1-a)\frac{c_3}{c_1}}{1+(1-a)\frac{c_3}{c_1}} y_k. \tag{7}$$

Equation (7) corresponds, in principle, to Eq. (2), which in turn is the basis of Eckhardt's algorithm. The comparison of Eqs. (7) and (1), or more precisely the comparison of the coefficients of b_{k-1} and y_k in both equations, yields

$$\frac{1}{1+(1-a)\frac{c_3}{c_1}} = \frac{1-\mathrm{BFI}_{\max}}{1-a\mathrm{BFI}_{\max}} \tag{8}$$

and

$$\frac{\frac{c_3}{c_1}}{1+(1-a)\frac{c_3}{c_1}} = \frac{\mathrm{BFI}_{\max}}{1-a\,\mathrm{BFI}_{\max}}. \tag{9}$$

The solution of this system of equations results in

$$\mathrm{BFI}_{\max} = \frac{c_3}{c_1 + c_3}. \tag{10}$$

In other words, a single assumption, namely that baseflow still begins at the same time step as precipitation, is sufficient to transform the algorithm of Furey and Gupta (2001) into the algorithm of Eckhardt (2005), where Eq. (10) holds.

3 Discussion

Eckhardt's algorithm represents a whole class of recursive digital filters that only differ by the value of $\mathrm{BFI}_{\max}$. These are the filters that are based on the assumption that baseflow is runoff from a linear reservoir and that are constructed according to Eq. (2). For example, setting $\mathrm{BFI}_{\max} = 0.5$ yields the filter of Chapman and Maxwell (1996). Do these filter algorithms lack a physical basis? Section 2 should have made it clear that this is not the case. The algorithm of Eckhardt (2005) differs from the algorithm of Furey and Gupta (2001) only in the time delay assumed between precipitation and the exfiltration of baseflow into surface waters and in the fact that two parameters, c_1 and c_3, are combined into one, $\mathrm{BFI}_{\max}$.

3.1 Time delay

Furey and Gupta (2001) introduced the parameter d in Eq. (5) as the number of time steps between precipitation and groundwater recharge. A sensitivity analysis they conducted showed that the filter performance was "relatively insensitive to changes in d" so that $d=0$ seemed to be an acceptable choice. Furthermore, when using Eq. (1), it is assumed that not only the groundwater recharge but also the generation of baseflow still occurs in the same time step as precipitation. When assessing these prerequisites, two aspects should be considered:

1. The streamflow component calculated with Eq. (1) is usually likely to consist not only of groundwater but also of transient water sources, including interflow (Cartwright et al., 2014; Yang et al., 2021).

2. In this publication, the algorithm of Eckhardt (2005) is compared to the model ideas of Furey and Gupta (2001) on the formation of baseflow. It is not compared to the reality. If the baseflow calculated with Eq. (1) occurs in Furey and Gupta's model world at the same time step as precipitation, this does not necessarily mean that it also corresponds to a runoff component in the real world that occurs without a relevant time lag compared to precipitation.

3.2 Model parameters

c_1 is the ratio of overland flow to precipitation, and c_3 is the ratio of groundwater recharge to precipitation. Furey and Gupta (2001) propose a method to determine c_1 and c_3 using additional precipitation data. $\mathrm{BFI}_{\max}$ could then be calculated with Eq. (10).

$\mathrm{BFI}_{\max}$ could also be determined in another way. If the fraction on the right-hand side of Eq. (10) is expanded with the precipitation, the result is

$$\mathrm{BFI}_{\max} = \frac{\text{groundwater recharge}}{\text{overland flow} + \text{groundwater recharge}}.$$

If one assumes that (a) there is no inflow or outflow of groundwater below the surface boundaries of the catchment and that (b) there is no evapotranspiration from groundwater or surface waters, then the sum of overland flow and groundwater recharge corresponds approximately to the streamflow:

$$\mathrm{BFI}_{\max} \approx \frac{\text{groundwater recharge}}{\text{streamflow}}. \tag{11}$$

Streamflow is given. Consequently, "only" a method for estimating mean groundwater recharge is needed to approximate $\mathrm{BFI}_{\max}$.

4 Conclusions

The recursive digital filter of Eckhardt (2005) largely coincides with the physically based algorithm of Furey and Gupta (2001). As Eckhardt (2005) has pointed out, his filter is identical to the filter of Boughton (1993) and passes for different values of the parameter $\mathrm{BFI}_{\max}$ in one-parameter filters like the one of Chapman and Maxwell (1996). Thus, the question posed in the title of this paper can justifiably be answered for a whole family of recursive digital filters with the following: yes, they are physically based.

The preceding considerations also suggest a way in which the parameter $\mathrm{BFI}_{\max}$ of Eckhardt's filter could be determined objectively, namely via groundwater recharge. Since the results of Eckhardt's filter are less sensitive to the parameter $\mathrm{BFI}_{\max}$ than to the parameter a (Eckhardt, 2012), the estimate for $\mathrm{BFI}_{\max}$ would not even have to be particularly accurate. The sensitivity of the baseflow index BFI to the parameter $\mathrm{BFI}_{\max}$ can be described by the sensitivity index

$$S(\mathrm{BFI}|\mathrm{BFI}_{\max}) = \frac{(a-1)(a\mathrm{BFI}-1)}{(1-a\mathrm{BFI}_{\max})^2}\,\frac{\mathrm{BFI}_{\max}}{\mathrm{BFI}} \tag{12}$$

(Eckhardt, 2012; Eq. 15). For sixty perennial streams with porous aquifers, Eckhardt (2012) has found a mean sensitivity index of 0.26. That is, a relative error of x percent in $\mathrm{BFI}_{\max}$ would result in a relative error of 0.26 times x percent in BFI. Thus, even if $\mathrm{BFI}_{\max}$ had an uncertainty of up to about 40 %, this would probably produce an uncertainty of, at most, 10 % in the calculated baseflow index.

Data availability. No data were re-evaluated for the present study.

Competing interests. The author has declared that there are no competing interests.

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