

# Supplement S2

July 12, 2022

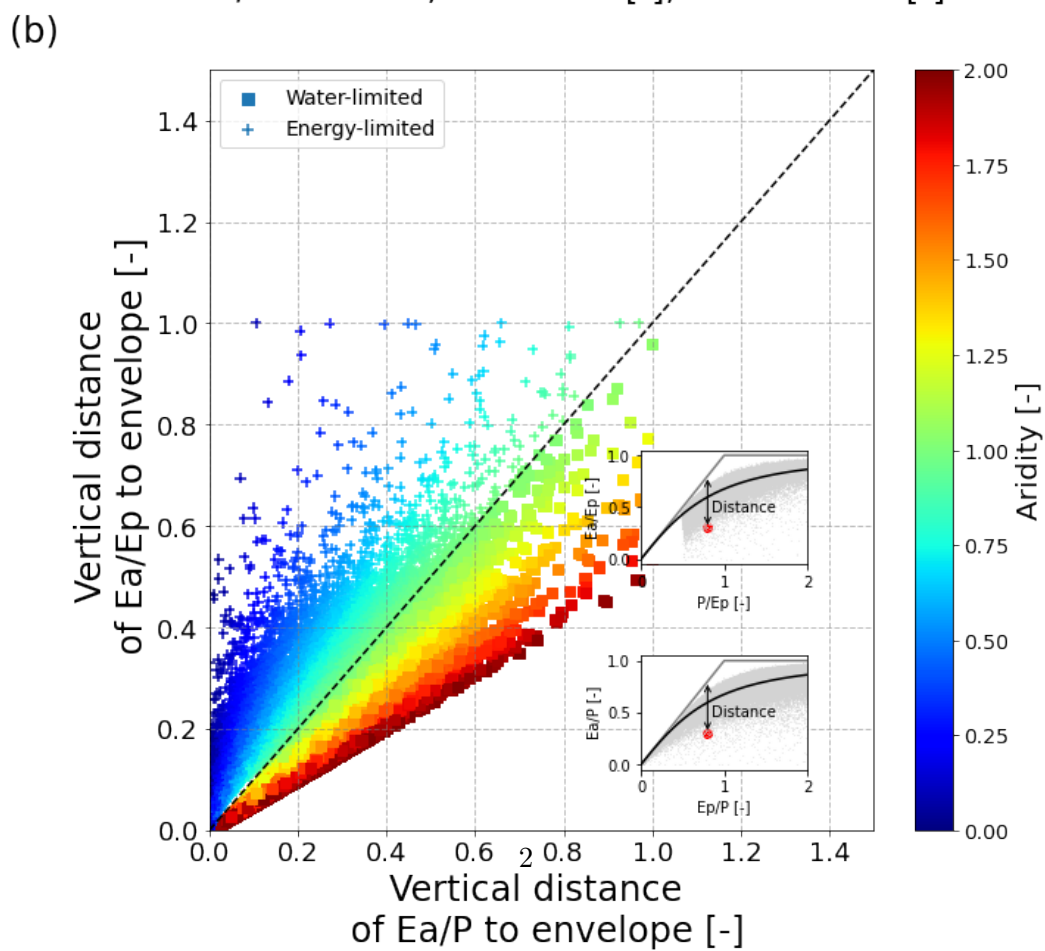
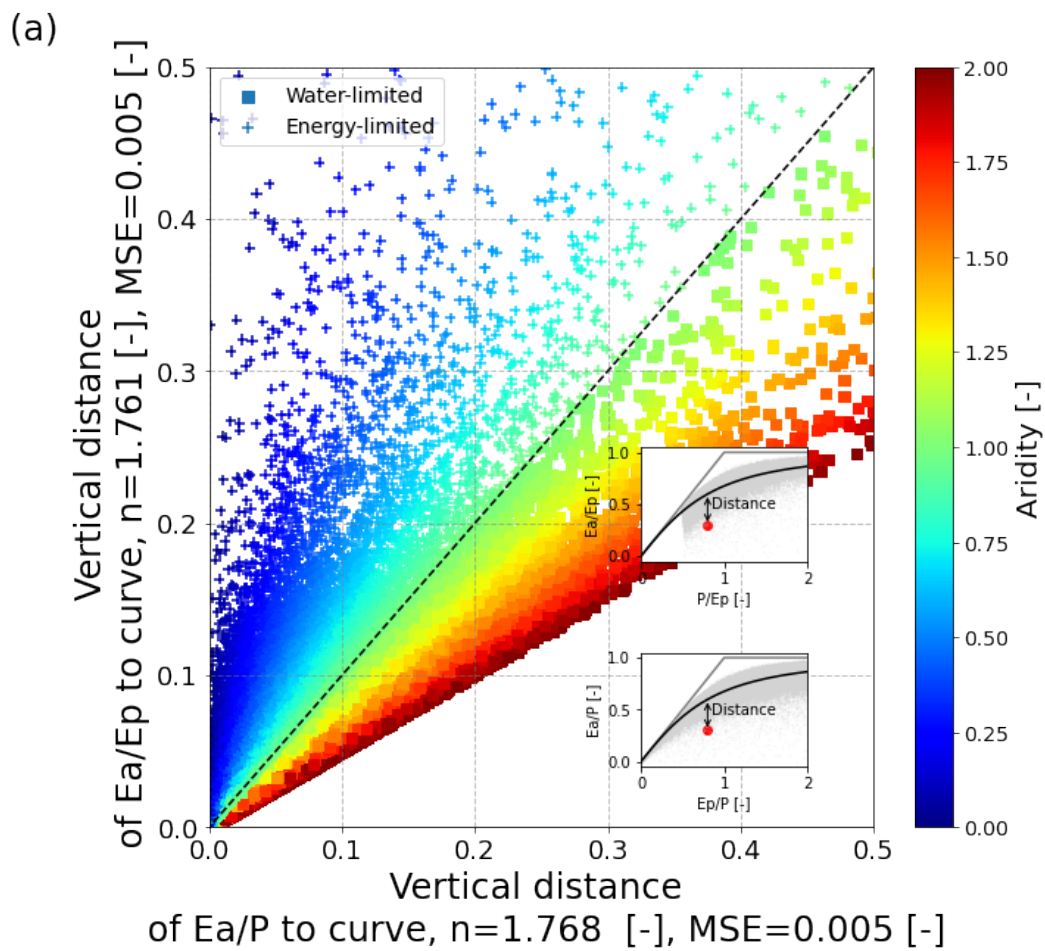
## 1 Distances to envelope and curve in a random Budyko space

Here, a random Budyko space was created, in order to determine more generally distances to the envelope and curve. This was done in the following way:

- $E_p/P$  values were samples from a uniform distribution with 100,000 samples between a value of 0 and 2.
- Accompanying  $n$ -values were sampled for each realization of  $E_p/P$  from a truncated normal distribution, truncated at 0.0 to avoid negative  $n$ -values, with a mean of 1.9 and standard deviation of 0.5 .
- Accompanying  $E_a/P$  and  $E_a/E_p$  values were calculated by using the Budyko equation with a certain realization of  $n$ .
- A Budyko curve was fitted to this data set
- Distances to the curve and envelope were determined

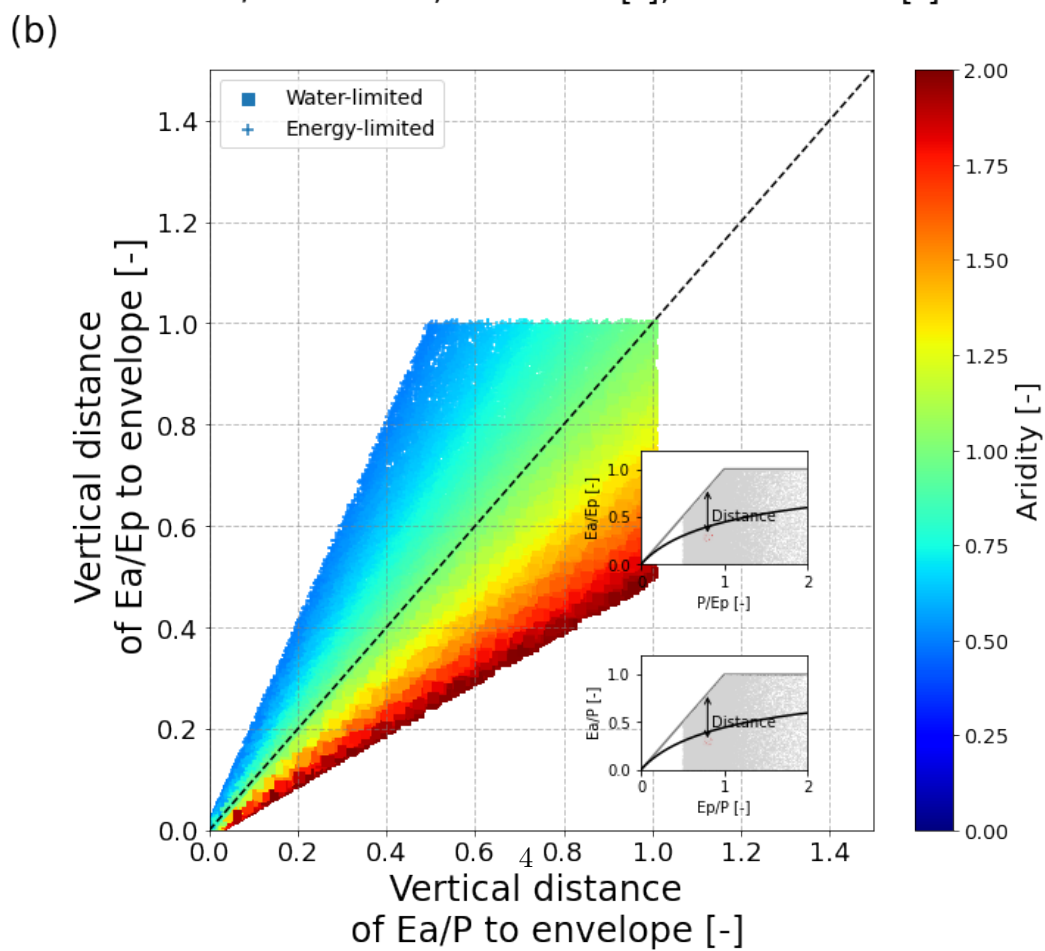
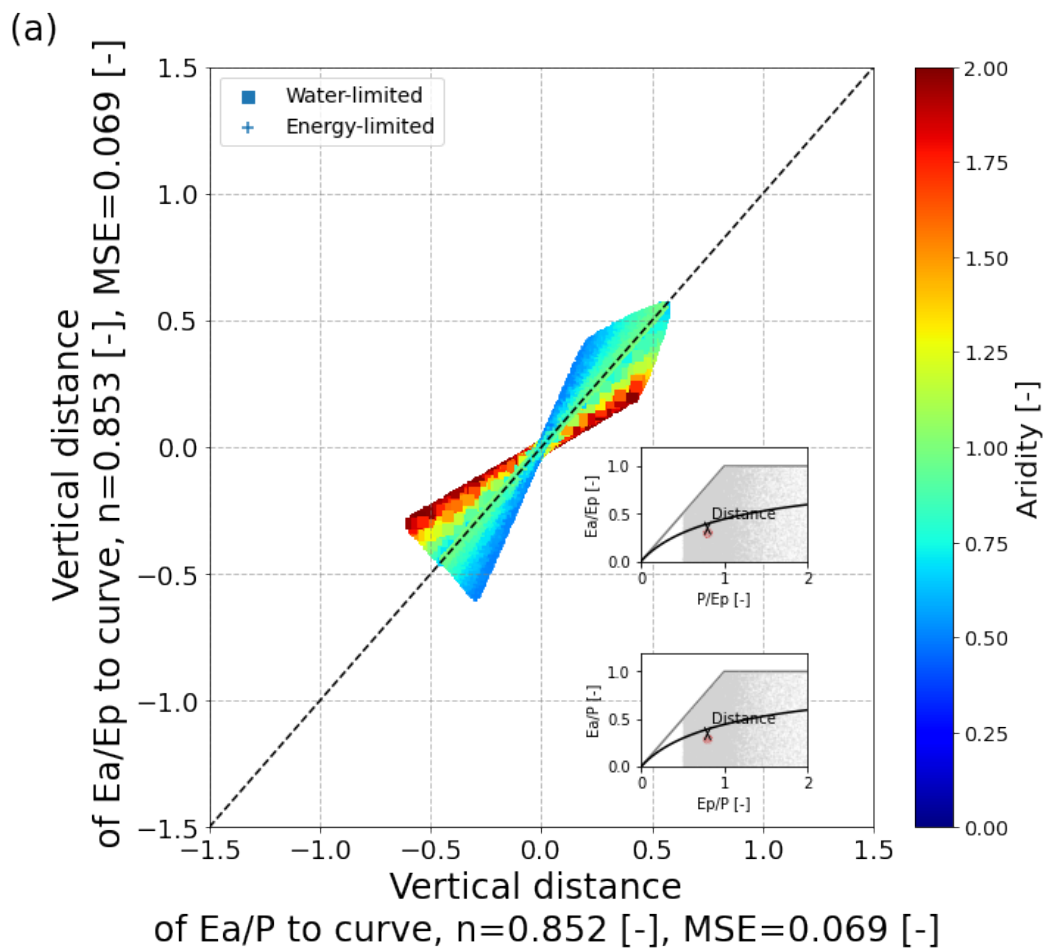
Similarly, a second random Budyko space was created:

- $E_a$ ,  $E_p$  and  $P$  were sampled 100000 times from a uniform distribution with values between 0 and 2000 mm/year.
- Unrealistic combinations of  $E_a$ ,  $E_p$  and  $P$ , were removed (e.g.  $E_p/E_a > 1.0$ )
- Samples with an aridity  $>2$  and  $< 0.5$  were removed.
- A Budyko curve was fitted to this data set
- Distances to the curve and envelope were determined



**Figure S2.1.** Vertical distances to a) the envelope of the physical limits of the Budyko framework and b) vertical distances to the fitted Budyko curve, both for projections normalized by precipitation (x-axes) and potential evaporation (y-axes).  $Ep/P$ -data is sampled from a uniform distribution with values between 0.0 and 2.0,  $n$ -values are sampled from a truncated normal distribution with a mean of 1.9 and standard deviation of 0.5, the distribution was truncated at 0. 100,000 data points were sampled. Water-limited catchments ( $Ep/P > 1$ ) are shown with stars, whereas energy-limited catchments are shown with crosses. The colorscale indicates the aridity of the catchments.

Remaining samples:  
24937



**Figure S2.2.** Vertical distances to a) the envelope of the physical limits of the Budyko framework and b) vertical distances to the fitted Budyko curve, both for projections normalized by precipitation (x-axes) and potential evaporation (y-axes).  $E_p$ ,  $P$  and  $E_a$  data are sampled from a uniform distribution with values between 0.0 and 2000.0 mm/year. Originally, 100,000 samples were drawn, but unrealistic combinations of  $E_p$ ,  $P$  and  $E_a$  were removed leading to 33446 samples. Water-limited catchments ( $E_p/P > 1$ ) are shown with stars, whereas energy-limited catchments are shown with crosses. The colorscale indicates the aridity of the catchments.