Algorithm 1 PINNs with domain decomposition

Step 0: Divide the spatial domain into $\Omega_U$ and $\Omega_L$ and assign two neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ to each domain. Determine the architecture of two neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ and weight parameters $\lambda_i$ in the loss function $L$ (Eq. 17).

Step 1: Construct the neural networks $\mathcal{N}_U(z, t; \Theta_U)$ and $\mathcal{N}_L(z, t; \Theta_L)$ with neural network parameters $\Theta_U$ and $\Theta_L$.

Step 2: Initialize the neural network parameters to $\Theta^0_U$ and $\Theta^0_L$.

Step 3: Given available data (e.g., initial and boundary conditions, measurement data), train the neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ by minimizing the loss function $L(\Theta_U, \Theta_L)$.

\[ i \leftarrow 0 \]

\[ \text{while } i < \text{max\_iteration\_Adam do } \]

\[ \Theta^{i+1}_U \leftarrow \Theta^i_U + \hat{\Theta}^i_U \]

\[ \Theta^{i+1}_L \leftarrow \Theta^i_L + \hat{\Theta}^i_L \]

\[ i \leftarrow i + 1 \]

\[ \text{end while} \]

\[ \text{while L-BFGS-B stopping criteria are not met do } \]

\[ \Theta^{i+1}_U \leftarrow \Theta^i_U + \hat{\Theta}^i_U \]

\[ \Theta^{i+1}_L \leftarrow \Theta^i_L + \hat{\Theta}^i_L \]

\[ i \leftarrow i + 1 \]

\[ \text{end while} \]