Algorithm 1 PINNs with domain decomposition

Step 0: Divide the spatial domain into Ω_U and Ω_L and assign two neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ to each domain. Determine the architecture of two neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ and weight parameters λ_i in the loss function $\mathcal{L}$ (Eq. 17).

Step 1: Construct the neural networks $\mathcal{N}_U(z,t;\Theta_U)$ and $\mathcal{N}_L(z,t;\Theta_L)$ with neural network parameters Θ_U and Θ_L.

Step 2: Initialize the neural network parameters to Θ_U^0 and Θ_L^0.

Step 3: Given available data (e.g., initial and boundary conditions, measurement data), train the neural networks $\mathcal{N}_U$ and $\mathcal{N}_L$ by minimizing the loss function $\mathcal{L}(\Theta_U,\Theta_L)$.

i $\leftarrow 0$

while $i <$ max_iteration_Adam **do**

$\Theta_U^{i+1} \leftarrow \Theta_U^i + \hat{\Theta}_U^i$

$\Theta_L^{i+1} \leftarrow \Theta_L^i + \hat{\Theta}_L^i$

i $\leftarrow$ i + 1

end while

while L-BFGS-B stopping criteria are not met **do**

$\Theta_U^{i+1} \leftarrow \Theta_U^i + \hat{\Theta}_U^i$

$\Theta_L^{i+1} \leftarrow \Theta_L^i + \hat{\Theta}_L^i$

i $\leftarrow$ i + 1

end while