Supplement of Hydrol. Earth Syst. Sci., 26, 445–467, 2022
https://doi.org/10.5194/hess-26-445-2022-supplement

Hydrology and Earth System Sciences
Open Access

Supplement of

Rediscovering Robert E. Horton's lake evaporation formulae: new directions for evaporation physics

Solomon Vimal and Vijay P. Singh

Correspondence to: Solomon Vimal (solomonvimal@ucla.edu)

Supplementary Material

A. A useful derivation of lake equilibrium levels drawn from Horton (1927)

The following derivation complements Eqn. (5) of the main text. Though it does not represent the same physical problem, it does provide an analogous solution of the same skeleton, from which the derivation steps of Horton for Eqn. (5) can be inferred. Our inference of the same is provided below the derivation.

Time taken for lake levels to stabilize after channel has been modified resembles the formulation of the x_c

Stage-discharge relationships for inflow and outflow of the lake are given by

$$Q = c + kh \qquad (A1)$$

Inflow, outflow and storage are related by,

$$Idt – Adh = Qdt \qquad (A2)$$

where

$$I = c + kh2 \qquad (A3)$$

Rearranging

$$(I - Q)dt = Adh \qquad (A4)$$

$$(c + kh2 - c - kh)dt = Adh \qquad (A5)$$

Reducing

$$k(h2 - h1)dt = Adh \qquad (A6)$$

$$dt = \frac{A}{k}\frac{dh}{h_2 - h} \qquad (A7)$$

The time it takes for lake level to reach a new mean equilibrium level from the time the change to the channel is made ($h = h_1, when\ t = 0$) is given by integrating the above,

$$\int dt = \int \frac{A}{k} \frac{dh}{h_2 - h} \Rightarrow t = -\frac{A}{k} log_e \frac{h_2 - h_1}{h_2 - h} \qquad (A8)$$

where h_1: water surface height at original mean stage (above improved channel bottom); h_2: water surface height at original stage (above original channel bottom); A: lake surface area.
Q: mean outflow rate; h: depth at time t referred to new control sill elevation; t: time taken for lake level to reach a new mean equilibrium level, time of change of channel t_0=0.

Inference of derivation vapor blanket horizontal distance considering Eqns. (A1-8): Analogous to the above derivation, Horton's derivation of Eqn. (5) must have been as follows: change of distance (dx) of vapor blanket disturbance is directly related to horizontal rate of change of vapor pressure and inversely related to the amount of vapor transported horizontally (m), and a constant related to elemental area (C) from which vapor is transported, as well as the VVPD. Rearranging and integrating by parts, and taking limits from x=0 to x_c, we get ratio of evaporation from windward to leeward sides, i.e. fringe of the lake where it is maximum to where it approaches a constant value at a distance x_c.

B. Fitting a function to Horton's wind velocity correction factor (w_0)

Several methods were tested in order to estimate the wind height at ground level as a function of measurement height and velocity at the given height, including: 1) Monkey Saddle; 2) shifted divergence in measurement height and root like behavior in velocity at that height; 3) multi-linear regression; and 4) polynomial regression (with and without log).

Monkey Saddle is given by,

$$z = ax^3 - bxy^2 + c \qquad (B1)$$

Root and shifted divergence is given by the shape,

$$z = ax^p(b + (y - cx - d)^{-q} \qquad (B2)$$

where, x: velocity measured at height H w_H; y: Height H; z: velocity at ground (at 1 foot height from surface, $w_{h=0}$); values of a, b, c, p, and q are 14.555, 0.05, 16.644, -68.614, 1.617, 0.65.
Substituting, the values of coefficients, the final equation is given by

$$w_0 = 14.555\, {w_H}^{1.617}(0.05 + (H - 16.614w_H + 68.614)^{-0.65} \qquad (B3)$$

Dr. Nikolai Mikuszeit, on Stack Overflow, provided a solution with an R^2 value of 0.999, which is given by

$$w_0 = \frac{1.874}{(H + 13.83)^{0.162}} \, w_H^{\left(\frac{0.949}{(H+1.228)^{0.052}}\right)} \qquad (B4)$$

C. Horton's updated bibliography – most comprehensive to our knowledge: titles include those of papers, reports, books, and technical discussions

S. No.	Year	Title
1	1896	A report for the New York State Engineer and Surveyor
2	1900	Computational works connected with hydraulic tests
3	1900	Report on the measurement of the volume of streams and the flow of water in the State of New York
4	1901	Available water power of Michigan and its economical development
5	1901	American canal problems with special reference to the state of New York
6	1902	The law of water as applied to paper mills
7	1903	Annual Report of the State Engineer and Surveyor of New York
8	1905	The drainage of ponds into drilled wells
9	1905	Snowfalls, freshets, and the winter flow of streams in the state of New York
10	1905	Progress of Stream Measurements for the Calendar Year 1904, Part 2, Hudson, Passaic, Raritan and Delaware River Drainages
11	1905	Report of progress of stream measurements for the calendar year 1904; Part VI, Great Lakes and St. Lawrence River drainage
12	1906	Surface drainage of land by tile
13	1906	Weir experiments, coefficients, and formulas
14	1906	Turbine water-wheel tests and power tables
15	1906	Underground water resources of Long Island, New York
16	1906	Report of progress of stream measurements for the calendar year 1905, Part II, Hudson, Passaic, Raritan, and Delaware River drainages
17	1906	Report of progress of stream measurements for the calendar year 1905; Part VI, Great Lakes and St. Lawrence River drainages
18	1906	Hudson, Passaic, Raritan, and Delaware River Drainages
19	1906	Great Lakes and St. Lawrence River drainages
20	1907	The Adirondack rainfall summit
21	1907	Weir experiments, coefficients, and formulas

22	1907	Determination of stream flow during the frozen season
23	1908	Deforestation, drainage and tillage with special reference to their effect on Michigan streams
24	1910	The Turbine Water Wheel as a Prime Mover
25	1911	Ebermayer's experiments on forest meteorology
26	1913	Effects of recent flood on New York streams; study of rainfall and stream discharge, with hydrographs for fourteen rivers
27	1913	Flood frequency and flood control
28	1914	Evaporation from snow and errors of rain gage when used to catch snowfall
29	1914	Discussion of Report of Committee on Yield of Drainage Areas
30	1914	Derivation of runoff from rainfall data. Discussion
31	1915	Idiosyncrasies of Underground Water
32	1915	The melting of snow
33	1915	Discussion of paper by A. F. Meyer on Computing Runoff from Rainfall and other Physical data
34	1915	Discussion: Yield of Underground Reservoirs
35	1916	Standing-wave experiment
36	1916	Some better Kutter's formula coefficients
37	1916	Diagram for full comparison of hydraulic turbines
38	1916	A study of the depth of annual evaporation from Lake Conchos, Mexico. . Discussion hy M. Hegly, Robert E. Horton, and J. W. Ledoux. p.
39	1917	A new evaporation formula
40	1917	A new evaporation formula developed
41	1917	Rational study of rainfall data makes possible better estimates of water yield
42	1917	Failure of hydraulic projects from lack of water prevented by better hydrology
43	1917	Determining the regulating effect of a storage reservoir
44	1917	Drainage Basin and Crop Studies Aid Water-Supply Estimates
45	1918	Air chimneys of ice below a waterfall
46	1918	Additional data needed by engineers
47	1918	Discussion on "obstruction to flow by bridge piers"
48	1919	Watershed Leakage in Relation to Gravity Water Supplies
49	1919	Additional meteorological data needed by engineers
50	1919	Evaporative capacity
51	1919	Device for obtaining maximum and minimum water surface temperatures

52	1919	Rainfall interception
53	1919	Some broader aspects of rain intensities in relation to storm-sewer design
54	1919	The measurement of rainfall and snow
55	1919	Discussion on The Duty of Water In the Pacific Northwest
56	1920	From the Committees: Hydrological Meteorology
57	1920	Modern hydraulic turbine design
58	1920	Comparison of snow-board and raingage-can measurements of snowfall
59	1920	Weather and literature
60	1921	Vapor pressure and humidity diagram
61	1921	Results of Evaporation Observations
62	1921	Correlation of maximum rain intensities for long and short time-intervals
63	1921	Discussion of the probable variation in yearly precipitation
64	1921	Cloudburst rainfall at Tarborton
65	1921	Unusual lightning
66	1921	Thunderstorm-breading spots
67	1921	The beginning of a thunderstorm
68	1921	The depletion of ground-water supplies
69	1922	Discussion of “The American Mixed-Flow Turbine”
70	1922	Discussion of "Siphon Spillways"
71	1923	Group distribution and periodicity of annual rainfall amounts
72	1923	Transpiration by forest trees
73	1923	Rainfall interpolation
74	1923	Accuracy of areal rainfall estimates
75	1923	Rainfall duration and intensity in India
76	1923	Discussion
77	1923	Engineering Meteorology and Hydrology
78	1924	Determining mean precipitation on a drainage basin
79	1924	Discussion on Distribution of Intense Rainfall
80	1924	The Distribution of intense Rainfall and some other Factors in the Design of Storm Water Drains
81	1924	Flood reduction by reservoirs
82	1924	Discussion of paper by C. S. Jarvis on flood flow characteristics
83	1927	Hydrology of the Great Lakes

84	1927	Report on the lake lowering controversy and a program of remedial measures
85	1928	Report on proposed tri-state compact [to] Board of Commissioners
86	1931	The field, scope and status of the science of hydrology
87	1931	Field, scope and status of hydrology, Water and Water Engineering
88	1931	New gravity water-supply system of Albany, N. Y.
89	1931	Discussion of "Horton on Regulation of Niagara River"
90	1932	Water diversion between drainage basins
91	1932	Drainage basin characteristics
92	1932	Discussion of the report of the committee on floods
93	1933	Slope table for fully controlled hydraulic experiments in open channels
94	1933	The relation of hydrology to the botanical sciences
95	1933	The role of infiltration in the hydrologic cycle
96	1933	Separate roughness coefficients for channel bottom and sides
97	1933	Storm-flow prediction
98	1933	Columnar Vapor Drift
99	1933	Primary Rainfall Types
100	1934	Water-losses in high latitudes and at high elevations
101	1934	Compilation and summary of the evaporation records of the Bureau of Plant Industry, U.S. Department of Agriculture, 1921-32
102	1934	Snow-surface temperature
103	1934	Laminar sheet flow
104	1934	Recent tendencies in relation to valuation of water rights
105	1934	Discharge coefficients for tainter gates
106	1934	Composite roughness in channels
107	1935	Surface runoff phenomena : Part I, Analysis of the hydrograph
108	1936	Natural stream-channel storage
109	1936	Maximum groundwater levels
110	1936	Surface-runoff control, Headwaters Control and Use, Chapter II
111	1936	Historical development of ideas regarding the origin of springs and ground water
112	1936	Relation of Hydraulic and Laboratory Resaerch to Physical and Economic Geography
113	1937	Hydrologic Interrelations of Water and Soils
114	1937	Determination of infiltration capacity for large drainage basins

115	1937	Hydrologic aspects of stream-flow stabilization
116	1937	Natural stream channel-storage (Second paper)
117	1937	Hydrologic aspects of stream-flow stabilization
118	1937	Hydrologic research
119	1938	Analysis of simulated rainfall experiments
120	1938	Channel waves subject chiefly to momentum control
121	1938	Phenomena of the contact zone between the ground surface and a layer of melting snow
122	1938	Rain wave-trains
123	1938	Seddon's and Forchheimer's formulas for crest velocity of flood-waves subject to channel-friction control
124	1938	Report on Soil Conservation Service special advisory committee, 1937-1938
125	1938	Definitions and classification of flood waves
126	1938	The interpretation and application of runoff experiments with reference to soil erosion problems
127	1938	Apples from Eden and other short stories
128	1939	Memorandum regarding purpose and procedure for research project on infiltration [in Delaware River]
129	1939	Analysis of runoff-plat experiments with varying infiltration capacity
130	1939	Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1938-1939
131	1939	What Can We Do About the Weather?
132	1940	Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1939-1940
133	1940	The infiltration-theory of surface-runoff
134	1940	Hydrophysical approach to quantitative morphology of drainage basins
135	1940	An approach toward a physical interpretation of infiltration capacity
136	1940	Suggestion for a comprehensive research program on runoff phenomena
137	1940	Delaware River Basin Flood Volumes, n. 1
138	1940	Determination of areal average infiltration-capacity from rainfall and runoff data
139	1940	Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils
140	1940	Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils
141	1941	The Role of Snow, Ice and Frost in the Hydrologic Cycle
142	1941	Flood-crest reduction by channel storage
143	1941	Sheet erosion: past and present

144	1941	Virtual channel-inflow graphs
145	1941	Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service
146	1941	Discussion (in response to N. E. Edlefsens, Report of the committee on the physics of soil-moisture, 1940-1941, pp. 917-926)
147	1941	Discussion (in response to M. R. Huberty and A. F. Pillsbury, Factors influencing infiltration-rates into some California soils, pp. 686-693)
148	1942	Discussion (in response to A. B. C. Anderson, J. E. Fletcher, and N. E. Edlefsen, Soil-moisture conditions and phenomena in frozen soils, pp. 356-364)
149	1942	Derivation of infiltration-capacity curve from infiltrometer experiments
150	1942	Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1941-1942
151	1942	Remarks on hydrologic terminology
152	1942	An experiment on flow through a capillary tube
153	1942	Closure to discussion (in response to Horton, R. E., An experiment on flow through a capillary tube, pp. 534-538)
154	1942	Simplified method of determining an infiltration-capacity curve from an infiltrometer-experiment
155	1942	A simplified method of determining the constants of the infiltration-capacity equation
156	1942	Some effects of rain erosion and sedimentation on infiltration-capacity
157	1943	Evaporation—Maps of the United States
158	1943	Hydrologic interrelations between lands and oceans
159	1943	On the relation of soil conservation to air and ground-water pollution
160	1943	A discussion of the relation of soil conservation to air and ground-water pollution
161	1944	Report on proposed improvement and extension of Hemlock Lake water supply system, Rochester, N.Y
162	1944	Some Hydrologic Characteristics of the United States, Part 1
163	1945	Infiltration and runoff during the snow-melting season, with forest-cover
164	1945	Erosional development of streams and their drainage basins, hydrophysical approach to quantitative morphology
165	1947	Preliminary outline for a comprehensive research on runoff phenomena
166	1948	The physics of thunderstorms
167	1948	Statistical distribution of drop sizes and the occurrence of dominant drop sizes in rain
168	1949	Convectional vortex rings – hail

D. Tips to find Horton's papers and full bibliography:

1. Using data from the table provided above, perform a Google search term as follows: "$title + $year + "Robert E. Horton" (side note: we found it easy to save the full citation using Zotero's plugin for browsers);

2. Search in AGU's Virtual Hydrology bibliography list maintained at the website - https://connect.agu.org/hydrology/vhp-scope/roberthorton (accessed Nov. 1, 2021);

3. Check the online archive of Albion College (Horton's *Alma Mater,* see Accavitti, 2019);

4. Contact the corresponding author (Solomon Vimal) by email to check in his personal bibliography collection (access can be granted to an in-progress Google Sheet where notes on bibliography and the content and working website/download link are curated);

5. Go to the U.S. National Archives in Maryland and dig into the 94 boxes (see list of boxes in Beven, 2004a).

One of these 5 approaches, in the order presented, should help simplify the search for the full paper and citation.

E. Guide to use U.S. and metric equivalent equations for pan, small and large lakes

Required steps	**Units and notes**	**Pan evaporation**	**Small lakes & ponds**	**Large lakes**
Near ground wind velocity corrections	Horton's original formulation	Equation 3b	Equation 3b	Equation 3b
	Metric equivalents	Equation 3c	Equation 3c	Equation 3c
Wind Factor	Horton's original formulation	Equation 2b	Equation 2b	Equation 2b
Evaporation Formula	NA	1a	1b	1c
Correction for vapor blanket influence?	NA	Yes, Eq. 2b is essential	Yes, Eq. 2b if the size is within ~10 m in radius	No
Area factor correction?	NA	Yes, Eq. 4a-d	Yes, Eq. 4a-d	No
Correction for pan geometry	Horton's units	Yes, Eq. 2c	No	No
Convection correction	Calibration is necessary	Yes, Eq. 2b	Yes, Eq. 2b	Yes, Eq. 2b
Barometric pressure correction	NA, it is a ratio	Yes, Eq. 8c	Yes, Eq. 8c	Yes, Eq. 8c