



The relationship between  $\alpha$ ,  $\beta$  and  $\varepsilon$  is most clearly explained by example ( $P$ =precipitation):

1. Say at this point and time we have 3  $P$  estimates from different data products: 5 mm, 8 mm and 10 mm. We can calculate the standard deviation  $DIU = \text{SD}(5,8,10) = 2.52 \text{ mm}$

2. Assume also that we have 3 models for predicting  $X$ =runoff:

- *Model 1* assumes runoff is equal to  $2 \text{ mm d}^{-1}$  plus an exponential contribution from  $P$  if it exceeds 4 mm.
- *Model 2* is a very basic model, assuming constant runoff at this location based on the historical average, say 8.2 mm.
- *Model 3* assumes runoff is 50% of  $P$  plus a contribution from groundwater return flow that ranges from 0.1 mm to 100.0 mm depending on the state of belowground aquifers.

Driving our models with those  $P$  numbers to produce an estimate of  $X$ , we might get a table like this:

$P$ estimate	Runoff ( $\text{mm d}^{-1}$ ) from			SD across models ( $\text{mm d}^{-1}$ )
	Model 1	Model 2	Model 3	
5 mm	$2.0 + \exp(5 - 4.0) = 4.7$	8.2	$(0.50 * 5) + 0.1 = 2.6$	2.8
8 mm	$2.0 + \exp(8 - 4.0) = 56.6$	8.2	$(0.50 * 8) + 10.0 = 14.0$	26.4
10 mm	$2.0 + \exp(10 - 4.0) = 405.4$	8.2	$(0.50 * 10) + 100.0 = 105.0$	207.1
SD across products ( $\text{mm d}^{-1}$ )	217.9	0.0	56.1	Mean from the left = $91.3 \text{ mm d}^{-1}$ Mean from above = $78.8 \text{ mm d}^{-1}$

3. Note that  $DOU = \text{mean}(\text{SDs across products}) = 91.3 \text{ mm d}^{-1}$ , which is not equal to  $MU = \text{mean}(\text{SDs across models}) = 78.8 \text{ mm d}^{-1}$  (there is no constraint for these to be equal in general). We are interested in when these values are greater or less than  $DIU$ , so we consider the scaled uncertainties  $\alpha = (DOU \div DIU)$  and  $\beta = (MU \div DIU)$ .

4. Note the key difference between  $\alpha$ , which is calculated from the outputs of the model, and  $DIU$ , which is calculated from the inputs: why not just consider  $DIU$ ? Because our focus is on  $X$  and therefore we need to quantify the *uncertainty introduced into  $X$  by the precipitation* ( $\alpha$ ), which is not the same as the uncertainty in the precipitation ( $DIU$ ) (this is an attribution study, therefore we focus on  $\alpha$  rather than  $DIU$ ).

5. In this analysis, we considered SDs of extreme event occurrence (EE/yr) rather than SDs of straight  $X$  values, which we have done for two reasons: (i) this allows us to consider and compare consistently the uncertainties of different response variables with different units (e.g.  $X$ =runoff vs.  $X$ =evapotranspiration) and (ii) in a global analysis it is necessary to compare across biomes (e.g. a desert point with a rainforest point) and using event occurrence statistics avoids the bias towards wet or dry regions (because of their greater absolute values of e.g. runoff) that must be corrected for in studies that work with the absolute values of  $X$ . Using occurrence statistics doesn't change the calculations of  $\alpha$ ,  $\beta$  and  $\varepsilon$  above, but does involve the additional assumption of a baseline distribution against which we may measure how extreme conditions are (see §2.1).