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Supplement of

Interpretation of multi-scale permeability data through an information theory perspective

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Supplementary Material
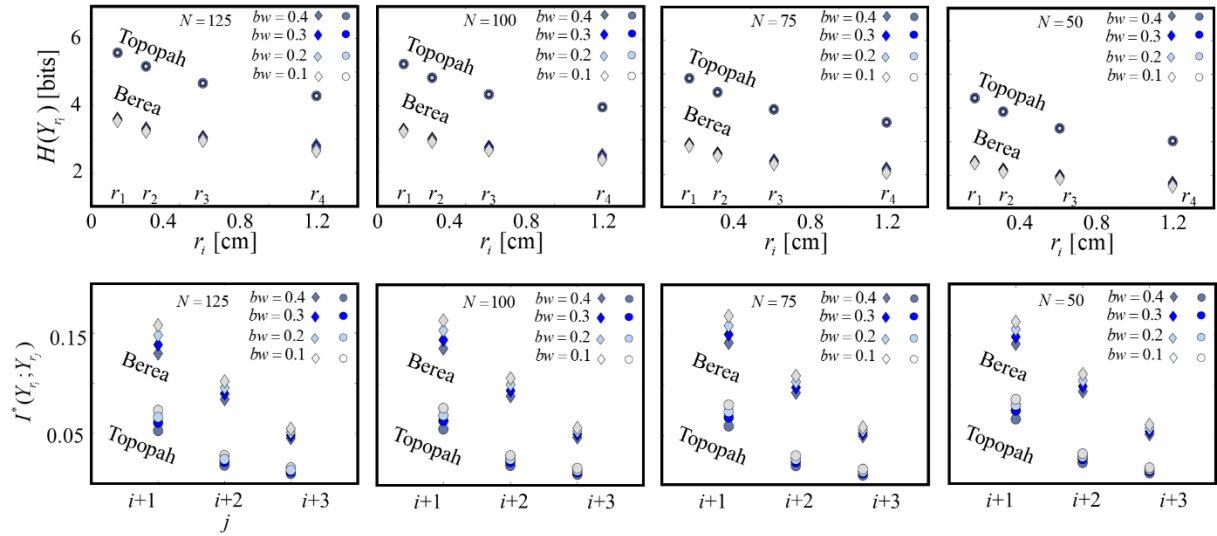
List of Figures included as Supplementary Material:

Figure S1. Shannon entropy ($H(Y_{r_i})$) versus r_i for the Topopah (circles) and the Berea (diamonds) samples for various numbers of bins ($N = [50, 75, 100, 125]$) and sizes of the kernel bandwidth ($bw = [0.1, 0.2, 0.3, 0.4]$). Bivariate normalized mutual information $I^*(Y_{r_j}; Y_{r_1}) = I(Y_{r_j}; Y_{r_1})/H(Y_{r_1})$ between data at the support scale r_1 (i.e., Y_{r_1}) and data at larger support scales, considering the Berea and the Topopah rock samples for various numbers of bins ($N = [50, 75, 100, 125]$) and sizes of the kernel bandwidth ($bw = [0.1, 0.2, 0.3, 0.4]$). Similar results are obtained considering data at the support scale r_2 (i.e., Y_{r_2}) and data at larger support scales.

Inspection of Figure S1 suggests that the values of the Shannon entropy ($H(Y_{r_i})$) and of the normalized bivariate mutual Information ($I^*(Y_{r_j}; Y_{r_1})$) slightly depend on the number of bins (N) and on the size of the bandwidth (bw) employed in the Kernel Density Estimator (KDE) procedure, while the overall patterns of $H(Y_{r_i})$ and $I^*(Y_{r_j}; Y_{r_1})$ do not exhibit substantial variations with N and bw.
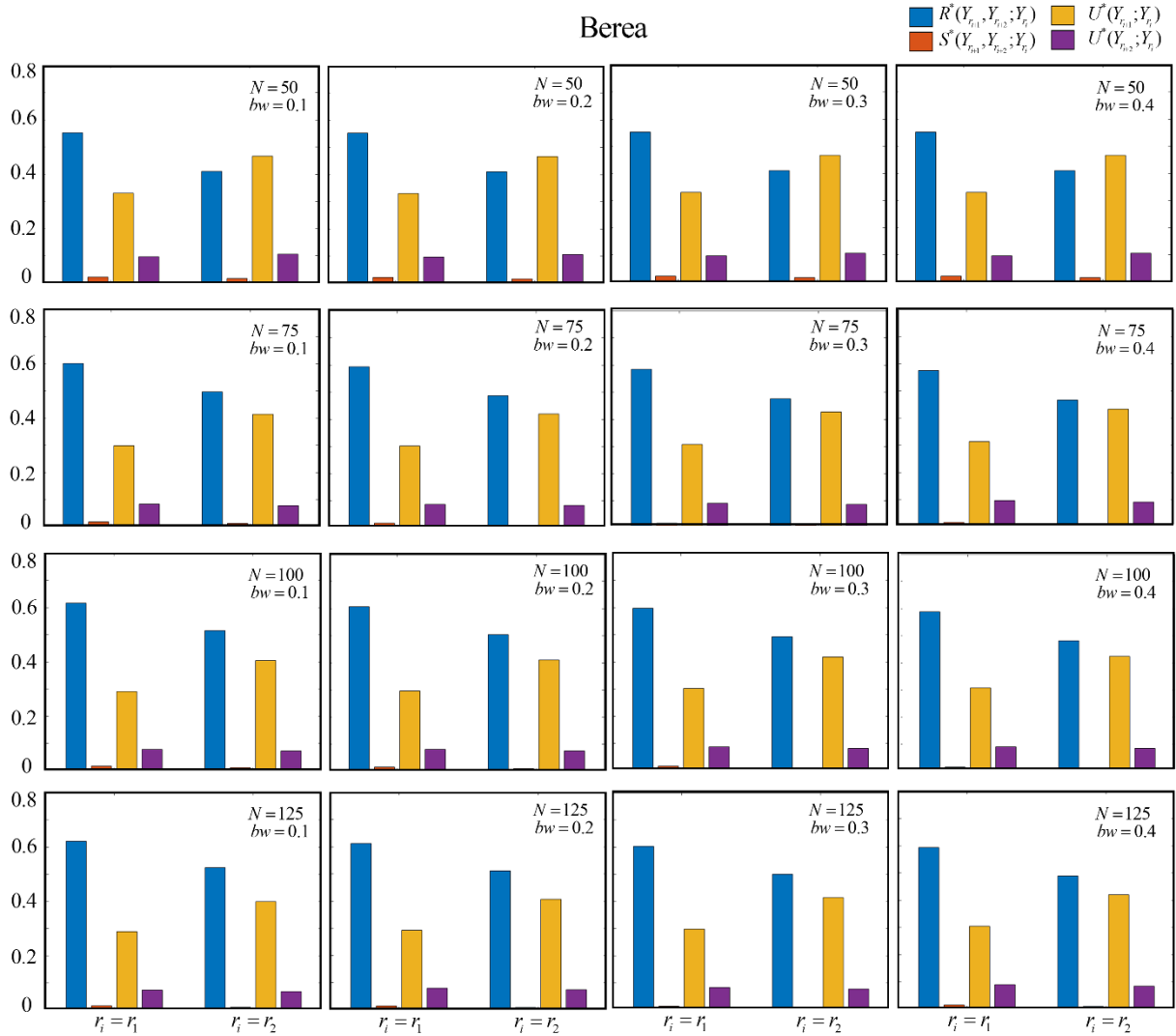
Figure S2. Information partitioning of the multivariate mutual information, $I(Y_{r_{i+1}}, Y_{r_{i+2}}; Y_{r_i})$, considering two triplets of data and $r_i = [r_1, r_2]$ for the Berea sample as a function of the number of bins ($N = [50, 75, 100, 125]$) and the size of the kernel bandwidths ($bw = [0.1, 0.2, 0.3, 0.4]$). For ease of comparison, we show the redundant, unique, and synergetic, contributions normalized by $I(Y_{r_{i+1}}, Y_{r_{i+2}}; Y_{r_i})$.

Inspection of Figure S2 suggests that the value of the information partitioning of the multivariate mutual information for the Berea sample slightly depends on the number of bins (N) and on the size of the bandwidth (bw) employed in the KDE procedure, while the overall patterns of the information partitioning for the Berea sample do not exhibit substantial variations with N and bw.
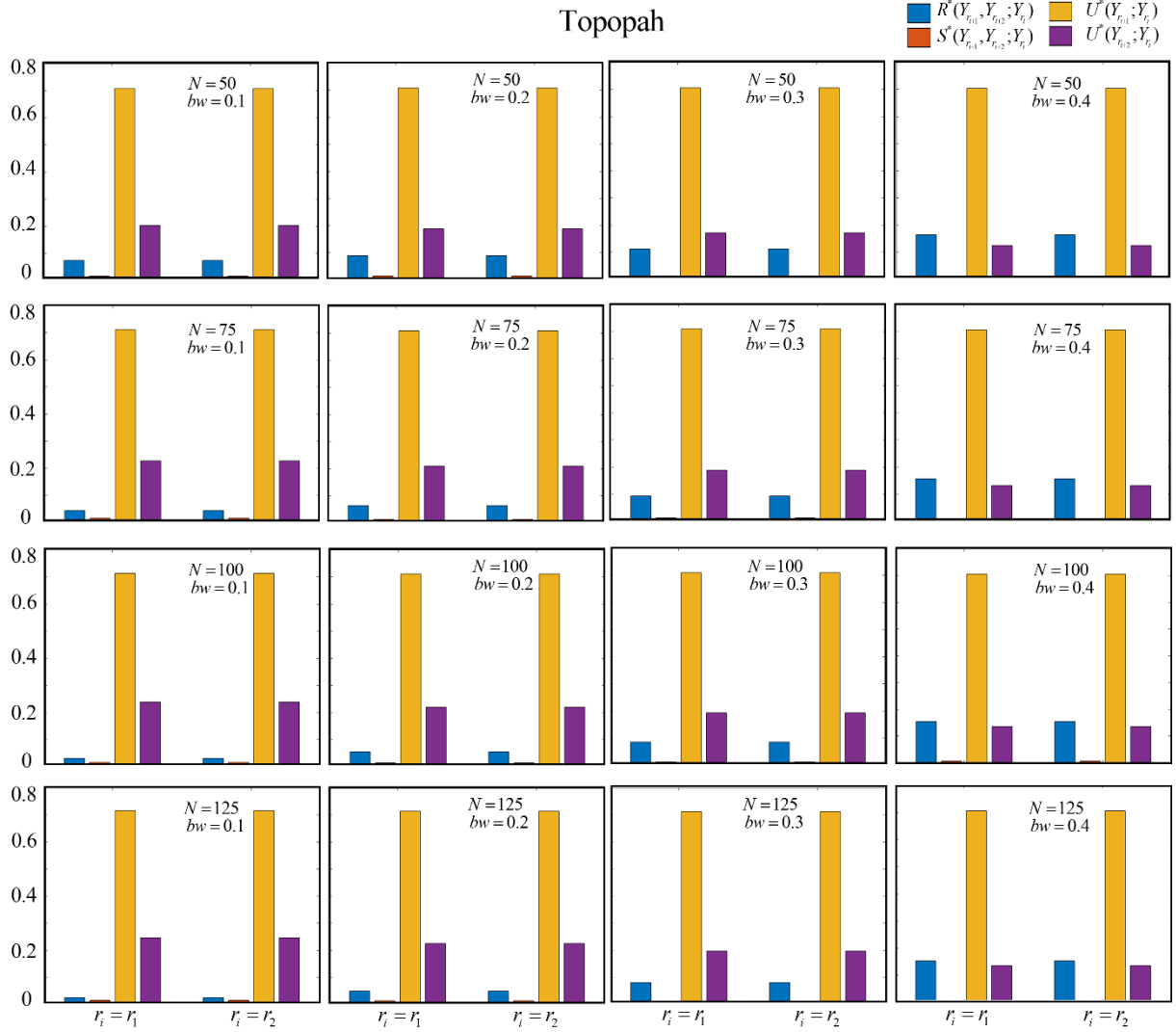
Figure S3. Information partitioning of the multivariate mutual information, $I(Y_{r_{i+1}}, Y_{r_{i+2}}; Y_{r_i})$, considering two triplets of data and $r_i = [r_1, r_2]$ for the Topopah sample as a function of the number of bins ($N = [50, 75, 100, 125]$) and size of the kernel bandwidth ($bw = [0.1, 0.2, 0.3, 0.4]$). For ease of comparison, we show the redundant, unique, and synergetic, contributions normalized by $I(Y_{r_{i+1}}, Y_{r_{i+2}}; Y_{r_i})$.

Inspection of Figure S3 suggests that the value of the information partitioning of the multivariate mutual information for the Topopah sample slightly depends on the number of bins (N) and on the size of the bandwidth (bw) employed in the KDE procedure, while the overall patterns of the information partitioning for the Berea sample do not exhibit substantial variations with N and bw.