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A line-integral-based method to partition climate and catchment effects on runoff

Mingguo Zheng

Correspondence to: Mingguo Zheng (mgzheng@soil.gd.cn)

Table S1 Data of the Batalling Ck

Year	R mm	P mm	E_0 mm
1979	8.156159	464.4431	1089.347
1980	19.64627	670.9428	1087.463
1981	42.57434	687.5864	1109.123
1982	53.15154	675.1814	1066.039
1983	79.30157	822.5461	1100.647
1984	40.90505	610.9478	1069.336
1985	46.05864	624.9977	1113.95
1986	18.24088	502.6219	1050.854
1987	9.915949	467.911	1110.182
1988	78.16621	771.4137	1189.523
1989	36.22554	626.2132	1078.752
1990	33.37795	631.4451	1028.017
1991	37.67235	688.0311	1109.712
1992	41.75197	743.0613	1009.535
1993	34.55473	613.4228	1043.555
1994	20.21549	472.8909	1159.976
1995	23.11371	656.5664	1125.72
1996	71.81743	833.4983	1141.496
1997	12.74668	505.7044	1093.114
1998	27.45874	714.279	1082.99
1999	24.45006	683.1997	1105.238
2000	35.67167	645.8951	1087.463
2001	6.404027	451.4155	1045.556
2002	24.55285	634.5722	1095.468
2003	29.86755	646.0289	1103.002
2004	31.36959	646.0731	1121.13
2005	41.08764	714.5904	1036.845
2006	5.214965	457.8624	1063.802
2007	25.29698	716.754	1068.393
2008	27.9267	641.0637	1012.949

Calculation steps

1) Data preparation and dividing the evaluation period into a number of subperiods

Table S2 Data preparation

Period	Subperiod	R	P	E_0	$E=P-R$	n	ΔR	ΔP	ΔE_0	Δn
Reference period	1979-1984	40.6	655	1087	614	3.0				
Subperiod 1	1985-1991	37.1	616	1097	579	2.9	-3.5	-39.2	10.3	-0.19

Subperiod 2	1992-1998	33.1	648	1094	615	3.2	-4.0	32.4	-3.5	0.38
Subperiod 3	1999-2008	25.2	624	1074	599	3.5	-7.9	-24.7	-19.8	0.22

2) Calculating ΔR_P, ΔR_{E_0}, and ΔR_n for Subperiod 1

① Determining the integral path L:

L is given by parametric equations: $P = \Delta P t + P_1$, $E_0 = \Delta E_0 t + E_{01}$, $n = \Delta n t + n_1$, $t \in [0,1]$. Using the data in Table S2, we get:

$$P = -39.2t + 655,\ E_0 = 10.3t + 1087,\quad n = -0.19t + 3.0,\quad t \in [0,1] \qquad \text{(S1)}$$

② Calculating ΔR_P using equations as follows:

$$\Delta R_P \approx 0.001 \Delta P \sum_{i=0}^{999} R_P(t_i) \quad t_i = 0.001i \text{ and } i \text{ is integer-valued} \qquad \text{(S2)}$$

$$\frac{\partial R}{\partial P} = R_P(P, E_0, n) = 1 - \frac{{E_0}^{n+1}}{(P^n + {E_0}^n)^{1/n}} \qquad \text{(S3)}$$

The equations above are the same as Eq. (6a) and (2a) in the manuscript, respectively. Substituting Eq. (S1), Eq. (S3) becomes a one-variable function of t, *i.e.*,

$$\frac{\partial R}{\partial P} = R_P(t) = 1 - \frac{(10.3t + 1087)^{(-0.19t+3)+1}}{[(-39.2t + 655)^n + (10.3t + 1087)^{(-0.19t+3)}]^{1/(-0.19t+3)}} \qquad \text{(S4)}$$

Then we can calculate $R_P(t_i)$ for each i in Eq. (S2) and we thus obtain:

ΔR_P for Subperiod 1 = 8.58

③ Using Eq. (S1) and Eqs. (6b-6c) and (2b-2c) in the manuscript, we can obtain ΔR_{E_0}, and ΔR_n in a similar way:

ΔR_{E_0} for Subperiod 1 =-0.95, and ΔR_n for Subperiod 1 =5.91

3) Repeating Step 2, we get ΔR_P, ΔR_{E_0}, and ΔR_n for each subperiod.

Table S3 Calculated ΔR_P, ΔR_{E_0}, and ΔR_n for each subperiod

	R	P	E_0	n	ΔR	ΔP	ΔE_0	Δn	ΔR_P	ΔR_{E_0}	ΔR_n
Subperiod 1	37.1	616	1097	2.9	-3.5	-39.2	10.3	-0.19	-8.6	-0.95	5.9
Subperiod 2	33.1	648	1094	3.2	-4.0	32.4	-3.5	0.38	6.6	0.3	-10.7
Subperiod 3	25.2	624	1074	3.5	-7.9	-24.7	-19.8	0.22	-4.5	1.6	-4.9

4) Summing up the last three columns in Table S3 respectively, we obtained the same results as in the line 2 in Table 3:

ΔR_P for the Batalling Ck = -6.49,

ΔR_{E_0} for the Batalling Ck = 0.95,

and ΔR_n for the Batalling Ck = -9.74

R codes

To calculate *n*

```
nCal<-function(P,E0, R) {
```

```
    Calculated_n<-c()
    n<-seq(0.1,15,0.01)
    for (i in 1:length(P)){
      Calculated_E<-MCY(P[i],E0[i],n=n)
      Calculated_R<-P[i]-Calculated_E
      Error<-abs(Calculated_R-R[i])
      t<-Error[2:length(Error)]-Error[1:(length(Error)-1)]
      tt<-sapply(t,function(x) ifelse(x<0,FALSE,TRUE))
      i<-which(tt)[1]
      Calculated_n<-c(Calculated_n,0.5*(n[i]+n[i+1]))
    }
    return(Calculated_n)
  }

  MCY<-function(P,E0, n) {
    x<-E0*P/(P^n+E0^n)^(1/n)
    return(x)
  }
```

To perform the LI method

```
LI<-function(P1,P2,E01,E02,R1,R2,m=0.001) {
  t<-seq(0,1,m)
  t<-t[-length(t)]
  P<-(P2-P1)*t+P1
  E0<-(E02-E01)*t+E01
  n1<-nCal(P=P1,E0=E01,R=R1)
  n2<-nCal(P=P2,E0=E02,R=R2)
  n<-(n2-n1)*t+n1
  PD_R<-PD_R(P=P,E0=E0,n=n)
  tt<-apply(PD_R,2,sum)
  LI<-c(m*(P2-P1)*tt[1],m*(E02-E01)*tt[2],m*(n2-n1)*tt[3])
  names(LI)<-c("C(P)","C(E0)","C(n)")
  return(LI)
}

PD_R<-function(P,E0, n) {
  PD_EtoP<-MCY(P,E0, n)/P*(E0^n/(P^n+E0^n))
  PD_EtoE0<-MCY(P,E0, n)/E0*(P^n/(P^n+E0^n))
  PD_Eton<-MCY(P,E0, n)/n*(log(P^n+E0^n)/n-(P^n*log(P)+E0^n*log(E0))/(P^n+E0^n))
  PD<-data.frame(1-PD_EtoP,-1*PD_EtoE0,-1*PD_Eton)
  names(PD)<-c("PD_R/P","PD_R/E0","PD_R/n")
  return(PD)
}
```

Table S4 Comparisons of the path-averaged sensitivities with the point sensitivities of runoff [a, b]

Catchment NO.	$\overline{\lambda_P}$	$\overline{\lambda_{E0}}$	$\overline{\lambda_n}$	λ_P	λ_{E0}	λ_n
1	0.68	-0.55	-17	0.621	-0.39	-71.8
2	0.2	-0.08	-27.3	0.227	-0.1	-30.9
3	0.58	-0.36	-26.7	0.68	-0.42	-79
4	0.3	-0.16	-30.5	0.39	-0.2	-50.1
5	0.33	-0.14	-43.1	0.394	-0.19	-59.4
6	0.29	-0.16	-26.5	0.352	-0.2	-34.9
7	0.71	-0.32	-223	0.781	-0.33	-299
8	0.49	-0.26	-77.9	0.478	-0.27	-64.9
9	0.16	-0.07	-11.8	0.161	-0.07	-17.6
10	0.72	-0.45	-57.3	0.74	-0.53	-61.1
11	0.25	-0.15	-19.8	0.29	-0.17	-22.5
12	0.34	-0.18	-37.2	0.393	-0.21	-48.6
13	0.68	-0.22	-275	0.719	-0.25	-303
14	0.7	-0.23	-326	0.745	-0.24	-378
15	0.66	-0.19	-320	0.708	-0.2	-378
16	0.65	-0.19	-315	0.692	-0.19	-363
17	0.58	-0.17	-153	0.602	-0.17	-175
18	0.32	-0.12	-50.1	0.402	-0.16	-69.6
19	0.2	-0.06	-29.2	0.234	-0.09	-34

[a] $\overline{\lambda_P}$ (10^{-3}m10^{-3}m^{-1}), $\overline{\lambda_{E0}}$ (10^{-3}m 10^{-3}m^{-1}), and $\overline{\lambda_n}$ (dimensionless) represent the path-averaged sensitivities of runoff to precipitation, potential evaporation, and catchment properties. If the evaluation period comprised only one subperiod, $\overline{\lambda_P}$, $\overline{\lambda_{E0}}$ and $\overline{\lambda_n}$ was calculated as: $\overline{\lambda_P} = \Delta R_P / \Delta P$, $\overline{\lambda_{E0}} = \Delta R_{E0} / \Delta E_0$,and $\overline{\lambda_n} = \Delta R_n / \Delta n$. If the evaluation period comprised N>1 subperiods, the equations became: $\overline{\lambda_P} = \sum_{i=1}^{N} |\Delta R_{Pi}| / \sum_{i=1}^{N} |\Delta P_i|$, $\overline{\lambda_{E0}} = -\sum_{i=1}^{N} |\Delta R_{E0i}| / \sum_{i=1}^{N} |\Delta E0i|$, and $\overline{\lambda_n} = -\sum_{i=1}^{N} |\Delta R_{ni}| / \sum_{i=1}^{N} |\Delta n_i|$, where the subscript i denotes the ith subperiod.

[b] λ_P , λ_{E0} , and λ_n represent the point sensitivities of runoff of the total differential method, which was calculated by substituting the observed mean annual values of the reference period into Eq. (2).