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Supplement of

Quantifying new water fractions and transit time distributions using ensemble hydrograph separation: theory and benchmark tests

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5 Formatting constraints required typesetting the matrices in Eqs. (40), (44), and (B7) with their lines wrapped, distorting their normal row and column organization. These equations are reproduced below in full-page width, allowing their matrices to be presented without line-wrapping.

Equation (40), Section 4.2

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_m \end{pmatrix} = \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{X}_0) & \text{cov}(\mathbf{X}_0, \mathbf{X}_1) & \text{cov}(\mathbf{X}_0, \mathbf{X}_2) & \cdots & \text{cov}(\mathbf{X}_0, \mathbf{X}_m) \\ \text{cov}(\mathbf{X}_1, \mathbf{X}_0) & \text{cov}(\mathbf{X}_1, \mathbf{X}_1) & \text{cov}(\mathbf{X}_1, \mathbf{X}_2) & \cdots & \text{cov}(\mathbf{X}_1, \mathbf{X}_m) \\ \text{cov}(\mathbf{X}_2, \mathbf{X}_0) & \text{cov}(\mathbf{X}_2, \mathbf{X}_1) & \text{cov}(\mathbf{X}_2, \mathbf{X}_2) & \cdots & \text{cov}(\mathbf{X}_2, \mathbf{X}_m) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \text{cov}(\mathbf{X}_m, \mathbf{X}_0) & \text{cov}(\mathbf{X}_m, \mathbf{X}_1) & \text{cov}(\mathbf{X}_m, \mathbf{X}_2) & \cdots & \text{cov}(\mathbf{X}_m, \mathbf{X}_m) \end{pmatrix}^{-1} \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{Y}) \\ \text{cov}(\mathbf{X}_1, \mathbf{Y}) \\ \text{cov}(\mathbf{X}_2, \mathbf{Y}) \\ \vdots \\ \text{cov}(\mathbf{X}_m, \mathbf{Y}) \end{pmatrix},$$

Equation (44), Section 4.2

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_m \end{pmatrix} = \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{X}_0)_{(0,0)} & \frac{n_{x_0 x_1}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_1)_{(0,1)} & \frac{n_{x_0 x_2}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_2)_{(0,2)} & \cdots & \frac{n_{x_0 x_m}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_m)_{(0,m)} \\ \frac{n_{x_1 x_0}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_0)_{(1,0)} & \text{cov}(\mathbf{X}_1, \mathbf{X}_1)_{(1,1)} & \frac{n_{x_1 x_2}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_2)_{(1,2)} & \cdots & \frac{n_{x_1 x_m}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_m)_{(1,m)} \\ \frac{n_{x_2 x_0}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_0)_{(2,0)} & \frac{n_{x_2 x_1}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_1)_{(2,1)} & \text{cov}(\mathbf{X}_2, \mathbf{X}_2)_{(2,2)} & \cdots & \frac{n_{x_2 x_m}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_m)_{(2,m)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{n_{x_m x_0}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_0)_{(m,0)} & \frac{n_{x_m x_1}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_1)_{(m,1)} & \frac{n_{x_m x_2}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_2)_{(m,2)} & \cdots & \text{cov}(\mathbf{X}_m, \mathbf{X}_m)_{(m,m)} \end{pmatrix}^{-1} \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{Y})_{(0,y)} \\ \text{cov}(\mathbf{X}_1, \mathbf{Y})_{(1,y)} \\ \text{cov}(\mathbf{X}_2, \mathbf{Y})_{(2,y)} \\ \vdots \\ \text{cov}(\mathbf{X}_m, \mathbf{Y})_{(m,y)} \end{pmatrix},$$

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Equation (B7), Appendix B

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_m \end{pmatrix} = \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{X}_0) & \frac{n_{x_0 x_1}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_1) & \frac{n_{x_0 x_2}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_2) & \cdots & \frac{n_{x_0 x_m}}{n_{x_0}} \text{cov}(\mathbf{X}_0, \mathbf{X}_m) \\ \frac{n_{x_1 x_0}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_0) & \text{cov}(\mathbf{X}_1, \mathbf{X}_1) & \frac{n_{x_1 x_2}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_2) & \cdots & \frac{n_{x_1 x_m}}{n_{x_1}} \text{cov}(\mathbf{X}_1, \mathbf{X}_m) \\ \frac{n_{x_2 x_0}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_0) & \frac{n_{x_2 x_1}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_1) & \text{cov}(\mathbf{X}_2, \mathbf{X}_2) & \cdots & \frac{n_{x_2 x_m}}{n_{x_2}} \text{cov}(\mathbf{X}_2, \mathbf{X}_m) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{n_{x_m x_0}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_0) & \frac{n_{x_m x_1}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_1) & \frac{n_{x_m x_2}}{n_{x_m}} \text{cov}(\mathbf{X}_m, \mathbf{X}_2) & \cdots & \text{cov}(\mathbf{X}_m, \mathbf{X}_m) \end{pmatrix}^{-1} \begin{pmatrix} \text{cov}(\mathbf{X}_0, \mathbf{Y}) \\ \text{cov}(\mathbf{X}_1, \mathbf{Y}) \\ \text{cov}(\mathbf{X}_2, \mathbf{Y}) \\ \vdots \\ \text{cov}(\mathbf{X}_m, \mathbf{Y}) \end{pmatrix}.$$