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Improvement of model evaluation by incorporating prediction and measurement uncertainty

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Table S1. The proof for the rationality of topological distance D.

objective	implementation	constraints
The definition of D	Defined $\forall f \in A$	the changing of it (*f*) is denoted by *g*;
The definition of g	$f'(x) = \begin{cases} 0 & (x < x_1) \\ f(x) & (x_1 \le x \le x_2) \\ 1 & (x > x_2) \end{cases}$	$\exists x_1, x_2 \in R,\ f(x) = 0$ and $f(x) = 1$, then $(x_1 < x_2)$;
	$f'(x) = \begin{cases} 0 & (x < x_1) \\ f(x) & (x \ge x_1) \end{cases}$	$\exists\ x_1 \in R,\ f(x) = 0$ and $\exists !\ x_2 \in R,\ f(x) = 1$;
	$f'(x) = \begin{cases} f(x) & (x \le x_2) \\ 1 & (x > x_2) \end{cases}$	$\exists\ x_2 \in R,\ f(x) = 1$ and $\exists !\ x_1 \in R,\ f(x) = 0$;
	$f'(x) = f(x)$	$\exists !\ x_1, x_2 \in R\ ,\ f(x) = 0$ and $f(x) = 0$;
The definition of B	$B = \{h \mid h = g(f(x))\}, B \neq \emptyset$	h is a continuous function;
The definition of Topological distance D	$D = \int_a^b \lvert f_1 - f_2 \rvert$	$a < b, a, b \in R$, a and b are all real numbers[a];
	$\forall D \notin \left\{ D \middle\vert D = \int_a^b \lvert f_1 - f_2 \rvert d_x,\ \forall f_1, f_2 \in B \right\}$	$\exists m \in R; m > D$ (D is a limited value);
Proof of positive Definiteness	$D(j - j) = 0$	$\forall j \in B, \lvert j - j \rvert = 0$, then $D = \int_a^b \lvert j - j \rvert d_x = 0$;
Proof of symmetry	$D(j_1, j_2) = D(j_2, j_1)$	$\forall j_1, j_2 \in B$, then, $\lvert j_1 - j_2 \rvert = \lvert j_2 - j_1 \rvert$;
Trigonometric inequality	$\lvert j_1 - j_3 \rvert = \lvert j_1 - j_2 + j_2 - j_3 \rvert \le \lvert j_1 - j_2 \rvert + \lvert j_2 - j_3 \rvert$	$\forall j_1, j_2, j_3 \in B$;
	$D(j_1, j_3) \le D(j_1, j_2) + D(j_2, j_3)$	$\int_a^b \lvert j_1 - j_3 \rvert d_x \le \int_a^b (\lvert j_1 - j_2 \rvert + \lvert j_2 - j_3 \rvert) d_x = \int_a^b \lvert j_1 - j_2 \rvert d_x + \int_a^b \lvert j_2 - j_3 \rvert d_x$

[a] where the values of a and b are as far from the origin as possible, thus the functions are integrated over a limited interval, and there are only small differences between the results and the results integrated for the real numbers R.