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# The geomorphological unit hydrograph—a critical review

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## Abstract

The theory of the geomorphological unit hydrograph (GUH) is examined critically and it is shown that the inherent assumption that the operation of the drainage network may be modelled by a corresponding network of linear reservoirs so restricts the instantaneous unit hydrograph (IUH) shape that the effects of further restrictions, reflecting the constraints imposed by the geomorphological laws of the channel network, cannot easily be identified. Without such identification, the geomorphological unit hydrograph theory is untestable and must remain only a plausible hypothesis providing an indication of a two-parameter IUH whose shape and scale factors must still be related empirically to appropriate catchment characteristics.

## Introduction

The geomorphological unit hydrograph (GUH) arises in the context of attempting to relate the instantaneous unit hydrograph (IUH) of a catchment to the geometry of the stream network and, ideally, to some expression of flow velocity or stream length, so that the IUH may be synthesized from information contained on a topographical map or, by extension, from the general relations observed in the geometrical configurations of stream networks, relations known as the geomorphological laws. The concept is presented as an alternative to the traditional method of seeking empirical relations between measures of IUH scale and shape and appropriate catchment characteristics, by the regression of the former on the latter, and the use of such relations, once established, to predict the IUHs for ungauged catchments. In fact, however, there is no way in which the empiricism of the traditional method can be avoided. The geomorphological relations do not contain the necessary information to determine the scale of the IUH which, of course, is very much more important than the shape, nor can the scale be determined without recourse to a measured velocity or some observation of delay time on the actual catchment, or by establishing an empirical relationship with some catchment characteristic or combination of characteristics, such as catchment area and/or slope. In the absence of such a relationship for the scale, the GUH theory is, necessarily, restricted to being a theory of IUH shape—a matter of relatively little practical importance and one concerning which any hypothesis would be difficult to verify, due to the difficulty of obtaining reliable expressions of the IUH shape empirically.

The original papers on the geomorphological unit hydrograph (Rodriguez-Iturbe and Valdés, 1979; Gupta *et al.*, 1980) were expressed, quite unnecessarily, in probabilistic terms, the IUH becoming the frequency distribution of the arrival time at the outlet of a drop of water randomly precipitated on the catchment. Each channel element, similarly, was assumed to produce a random delay time expressed as an exponential distribution, and channels of one order were assumed to discharge into channels of equal or higher orders [in Strahler's (1952) sense] randomly, in proportions determined by the geomorphological ratios. This model was shown by Chutha and Dooge (1990) to be equivalent to representing the surface drainage system by a network of linear reservoirs having the same two-dimensional configuration as that of the actual network of channel elements, with some assumed, not observed, relationship between the reservoir coefficients and the channel elements of different order. Thus, the necessity for the expression of the GUH model in probabilistic terms was avoided and henceforth the GUH could be discussed in classical hydrological terms.

In these terms, the GUH theory attempts to relate the shape, as distinct from the scale, of the IUH to the catchment by making two assumptions, viz.,

- 1) The IUH of a network of channel elements is equivalent to that of a network of linear reservoirs (as the analogous network, corresponding in two dimensional configuration with the actual network of channel elements), which bears some prescribed relationship between the reservoir storage coefficients and the physical characteristics of the channel network. [The form

of the relationship prescribed varies between different authors. Some authors would have the reservoir coefficients proportional to the channel element lengths (Gupta *et al.*, 1980; Chutha and Dooge, 1990) and others would assume the reservoir coefficient to be equal to the channel element length divided by a representative velocity (Rodriguez-Iturbe and Valdés, 1979). The velocity and the constant of proportionality provide a measure of the GUH scale which is undefined by the theory and it is implied that such a measure would be determined from records or measurements or from established regression relationships with catchment characteristics].

- 2) The geomorphological laws restrict the IUH shape in a manner which persists through the analogy. The manner by which these restrictions operate can therefore be found by imposing the corresponding restrictions on the network of linear reservoirs and computing the resulting IUHs.

It is evident that the validity of the second assumption (the restriction of the IUH shape and its persistence in the analogy) depends critically on the validity of the first. If it can be demonstrated that the analogous network always yields an IUH of moderate positive skewness (resembling in general shape virtually all empirically derived unit hydrographs of short rainfall duration), such agreement alone would not provide evidence for the second assumption. Indeed, the closer the similarity of shape exhibited by IUHs obtained from different networks of linear reservoirs, the less the prospect of predicting the effects on IUH shape of variations in the network. This even applies to those restrictions imposed in response to the geomorphological relations of the stream network through the analogous network. Hence, it is of the greatest importance that the validity of the first assumption (that the analogous network responds like the physical one) be tested; yet this assumption is often made uncritically.

## The gamma distribution GUH

Chutha and Dooge (1990) showed (*inter alia*) that, for networks conforming to the geomorphological relations, the IUH shape for the corresponding network of linear reservoirs closely resembled the two-parameter gamma distribution

$$h(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} e^{-\left(\frac{t}{k}\right)} \quad (1)$$

where  $n$  is a dimensionless shape parameter, and the product  $nk$  is a scale parameter corresponding to the mean delay time of the IUH.

This form was suggested by Edson (1951) as a general IUH equation and later shown, by Nash (1957), to be the exact impulse response of a single cascade of equal linear reservoirs and, by Nash (1960), to be a good approxima-

tion to the IUH of a similar cascade of linear reservoirs not restricted to equality of the storage parameters.

It would be a mistake however to interpret the work of Chutha and Dooge (1990) as showing that the geomorphological laws, *per se*, provide the source of this restriction. It is part of the thesis of the present paper that the restriction derives mainly from the first rather than the second GUH assumption and, to this end, extensive simulation experiments were carried out.

## The simulation experiments

To demonstrate the extent to which the first GUH assumption alone determines the IUH shape, the work described in detail in the appendix was undertaken. Networks of linear reservoirs of different complexity were generated, as seen in Figs. 1 to 3. These have different 'levels' (as defined in the appendix) and random construction according to rules which tended to produce mainly tree-like structures, but without constraints related to the geomorphological ratios. Runoff from contributing areas was represented by random distributions of input at the node points and these were routed through the network to produce the IUH at the catchment outlet. Dimensionless shape factors,  $m_2$  and  $m_3$ , respectively the second and third dimensionless moments of the IUH, were found and plotted in Figs. 5 to 9. It is evident from these figures that the points so obtained, each corresponding to a different simulation, tend to lie close to the curve corresponding to the gamma distribution, indicating that the IUHs of branching systems of linear reservoirs constructed randomly, but without any restriction deriving from the geomorphological laws of channel networks, exhibit convergence similar to that observed by Chutha and Dooge for the IUH of networks, in which such restrictions were imposed. Two consequences would seem to follow:

- 1) Convergence of the IUH shape factors to the limited regions indicated by figures 5 to 9 is a consequence of the first, rather than the second assumption of the GUH theory.
- 2) Because of this convergence, any real effects on the IUH shape imposed by the geomorphological ratios can be transmitted by the GUH model only insofar as these lie within the restrictions imposed by the first GUH assumption. That is, the effects of the geomorphological restrictions can be shown on the shape factor diagrams only as a further restriction of the limited regions.

It is, therefore, necessary to test the GUH assumptions very carefully. Unfortunately, such a test of a natural law relationship would be extremely difficult; evidence of necessity as well as sufficiency would be required. As a working hypothesis, however, providing the basis of a practical tool of applied hydrology, sufficiency alone would be required, i.e., does the model provide an adequate

expression of the IUH shape for use in a convolution with an effective rainfall record? In principle, a test of sufficiency might be attempted using two alternatives: directly by a one to one comparison of actual and model IUH shapes, or indirectly, by comparing the accuracy of reproduction of the observed flood hydrograph with a synthetic hydrograph obtained by convolution of the actual effective rainfall with the IUH whose shape is provided by the GUH theory and whose scale is determined from observation.

Direct comparison of actual and model IUH shapes would, however, be very difficult. Identification of the IUH from the rainfall-discharge records in which it is embedded involves de-convolution, a process which is as unstable as direct convolution is stable. The first casualty of such an operation would be the shape of the IUH. Furthermore, uncertainty of baseflow separation, definition of effective rainfall and neglect of non-uniform spatial distribution of rainfall all add to this difficulty. Experience suggests that IUHs derived from different events on the same catchment exhibit almost as much variation in shape as those obtained from events on different catchments. The indirect method of testing for sufficiency by comparing observed and computed flood hydrographs, would also prove difficult to apply because the process of convolution of the IUH and the effective rainfall so dampens out the effect of the IUH shape that significant differences in the hydrograph do not usually result from even substantial variations in IUH shape, as distinct from scale. For this very reason, of course, it is far more important that the scale, rather than the shape, be determined accurately when the IUH is used simply as a practical tool for generation of storm runoff.

## Discussion

Pending such tests of adequacy as a model of the IUH shape, and the very much more important matter of providing a satisfactory hypothesis for the IUH scale, the GUH theory must be regarded as an untested hypothesis. In the meantime, it would seem that little generality would be lost by assuming a two-parameter distribution (such as the gamma for the IUH) and the subsequent regression of the scale and shape parameters on appropriate catchment characteristics—that is, the traditional method.

The simulation results of Figs. 5–9 may, however, provide a suggestion of a hypothesis for IUH shape. Comparison of these figures indicates that, as the level of complexity of the simulated network increases from 2 to 6, the cluster of points representing the shape factors moves further down the gamma distribution curve towards the origin of the diagram, suggesting a relationship between catchment complexity and the IUH skewness.

## Conclusions

The theory of the geomorphological unit hydrograph does not provide a causal link between the geomorphological laws and the dynamic response of a catchment to rainfall. Crucially, the theory fails to address the matter of IUH scale and, in the matter of IUH shape, it relies critically on an untested hypothesis.

The indication of a restricted GUH shape, as demonstrated by the results of the simulation studies reported in the appendix, derives from the assumption that the dynamic operation of the catchment may be modelled by a network of linear reservoirs. The shape is, thus, not a unique consequence of the geomorphological constraints on stream network patterns.

As a basis for establishing an empirical relationship between the IUH and the catchment, the GUH theory would, in practice, be equivalent to the assumption of a general IUH equation described by a gamma distribution or similar functional form whose parameters must subsequently be related by regression (or otherwise) to appropriate catchment characteristics.

## Acknowledgements

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## Appendix

The construction of synthetic random networks of linear reservoirs of reasonably 'tree-like' shape is described herein. Dimensionless moments of the corresponding instantaneous unit hydrographs are obtained and are shown, through shape factor diagrams, to conform closely to those of the gamma distribution IUH, which is the exact impulse response of a single cascade of equal linear reservoirs. Thus, little extra generality of IUH shape is implied by the network compared with the single cascade of equal reservoirs and therefore little further restriction is imposed on the IUH by further restricting the network of reservoirs to reflect the geomorphological restrictions on the channel network.

### THE CONSTRUCTION OF RANDOM STREAM NETWORKS AND THEIR REPRESENTATIONS BY NETWORKS OF LINEAR RESERVOIRS

Fig. 1a shows a simplified stream network which forms the basis for constructing random stream networks with tree-like shapes.

In Fig. 1a, the network consists of three streams numbered (1), (2) and (3). The points shown by circles and labelled A, B and C are the nodes, points of origin or points at which two streams join to form a single stream. It is assumed that not more than two streams combine at

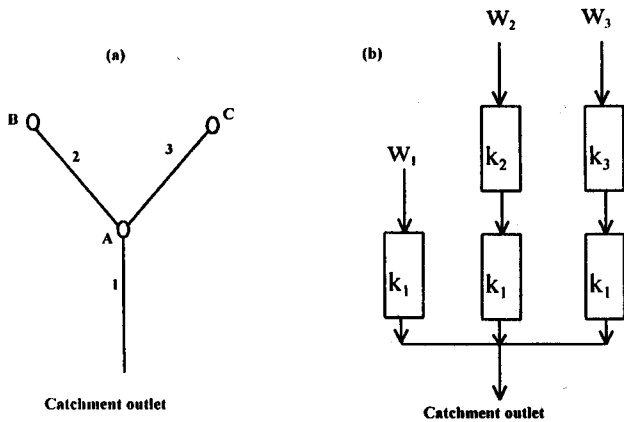


Fig. 1. A simple second level stream network and its representation by a network of linear reservoirs.

a single node. Each stream receives inflows at its upstream end from its own contributing area and/or from the feeding streams.

In the present work, each node is assigned a level equal to the number of streams through which drops of water starting at that node must pass before reaching the catchment outlet. Thus, nodes B and C are second level and node A is first level. The stream network is likewise designated by a level equal to the maximum of the node levels therein. Thus the network of Fig. 1a is second level.

If each stream of the network of Fig. 1a is represented by a linear reservoir of storage coefficient  $k_i$ , the network can be represented by three cascades of linear reservoirs connected in parallel (Fig. 1b);  $w_i$  denotes the contributing area for stream  $i$  ( $i = 1, 2, 3$ , in turn).

Third level stream networks can be constructed from second level networks by adding two extra streams at either or both second level nodes B or C. Addition of a

single stream at B or C would, of course, only locate the node further upstream without changing its level. Likewise, fourth level stream networks are constructed from the third level networks by adding extra streams (two per node) at the third level nodes and so on.

To illustrate how a third level network can be constructed from the second level network of Fig. 1a, assume that the extra streams are added at node C only. The resulting network is schematically shown in Fig. 2a. Two other possible third level configurations could be obtained by adding extra streams at node B, and at both nodes B and C.

Similarly, fourth level networks can be constructed from the network of Fig. 2a by adding extra streams at either or both third level nodes (i.e. E and F). Fig. 3a displays an example of one of the possible configurations of the fourth level network obtained by adding two streams to the network of Fig. 2a at node E.

Generally, the number of possible network configurations increases with the level of the network.

In the random construction used herein, networks of level  $(L)$  are constructed from those of the preceding level  $(L - 1)$ . In doing so, random decisions are made on how many, and at which of the  $(L - 1)$ -th level nodes the extra streams join. The level of the network is chosen *a priori*, the number of nodes  $n_d$  (from 1 to  $N_{L-1}$ ) to be expanded is chosen randomly with equal probability among the integers  $(1$  to  $N_{L-1})$ . The selection of  $n_d$  nodes is determined by random choice, each node not already chosen having equal probability of being chosen.

Networks so constructed tend to be well-conditioned or 'tree-like' but, exceptionally, unlikely configurations may also be obtained. To complete the network, it is necessary to assign weights ( $w$ ) proportional to the direct contributing areas of each stream element, and to determine the

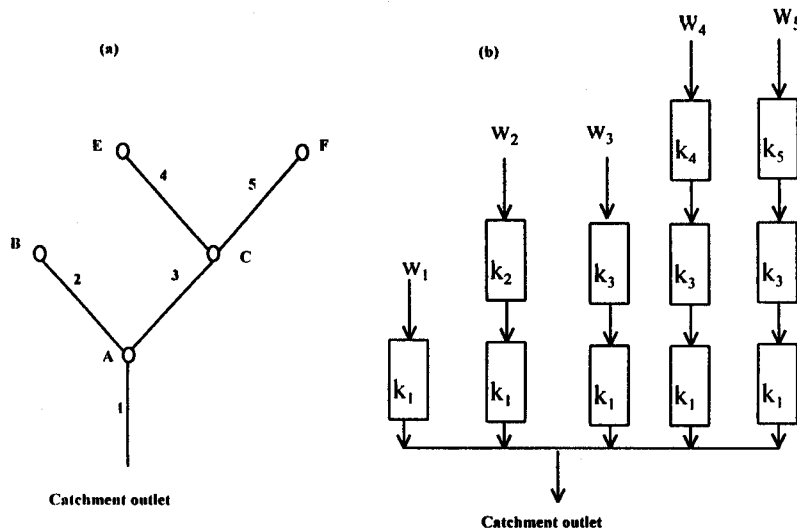


Fig. 2. An example of a third level stream network and its representation by a network of linear reservoirs. (Note that each path in Fig. 2(b) consists of a cascade of reservoirs, not necessarily equal, with inflow only at the upper end.)

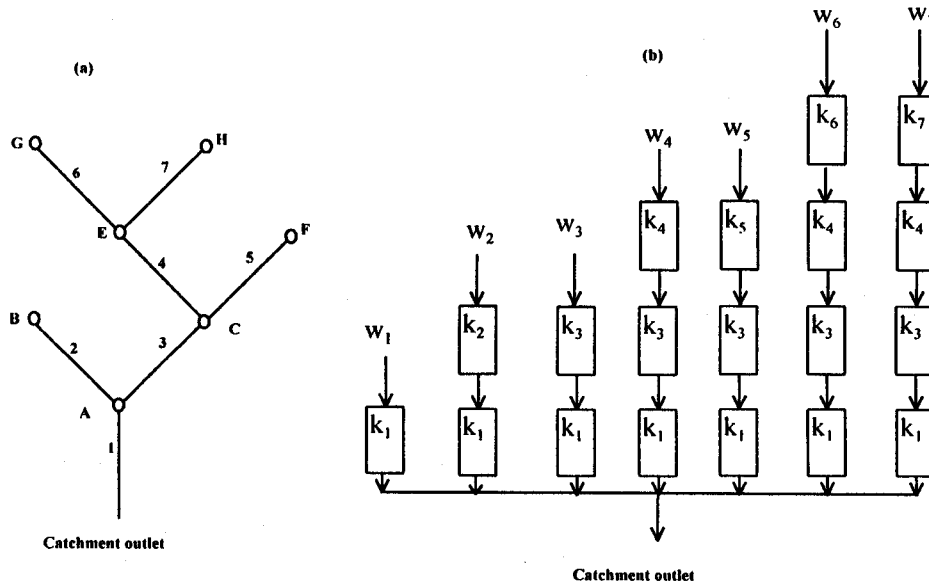


Fig. 3. An example of a fourth level stream network and its representation by a network of linear reservoirs.

storage coefficients  $k_i$ . Obviously, only relative weights and  $k$  ratios are required. For the contributing area ratios, a sequence of real positive random numbers is selected from a uniform distribution in the range  $[0,1]$ . These are then standardized by dividing each by the arithmetic sum of all. Likewise, the storage coefficient values, except for that of the first stream at the downstream end of the catchment which is taken as unity, are obtained by selecting a sequence of random numbers from a uniform range  $[0,1]$ . The effect of this is that the reservoir furthest downstream has a coefficient greater than any of the others in the network. This would seem a reasonable, though arbitrary, restriction for a network representing a catchment drainage system.

THE INSTANTANEOUS UNIT HYDROGRAPHS OF THE NETWORKS OF THE LINEAR RESERVOIRS

The IUH of each network of linear reservoirs is found by routing a delta function of unit volume through the network. The unit volume is distributed among the various pathways (each of which is a cascade of linear reservoirs) according to the relative contributing areas  $w_i$ . The IUH of the network is the weighted sum of the IUHs of the various pathways and can be expressed conveniently as

$$IUH(t) = \sum_{i=1}^{npath} w_i h_i(t) \quad (A 1)$$

where  $npath$  is the total number of pathways (i.e. the number of cascades) and  $h_i(t)$  is the IUH of the  $i$ -th cascade.

The  $r$ -th moment of the IUH about the origin is also the weighted sum of the  $r$ -th moment of the IUHs of the various pathways and can be expressed as

$$U_r' = \sum_{i=1}^{npath} w_i U_r' \quad (A 2)$$

where  $U_r'$  is the  $r$ -th moment of the impulse response function of the  $i$ -th pathway.

THE MOMENTS OF THE IUH

The moments about the time origin, or about the centre of area, provide useful descriptive parameters of the instantaneous unit hydrograph. The  $r$ -th moment  $U_r'$  of the IUH about the time origin is defined by

$$U_r' = \frac{\int_0^{\infty} t^r h(t) dt}{\int_0^{\infty} h(t) dt} \quad (A 3)$$

and the  $r$ -th moment  $U_r$  about the centre of area by

$$U_r = \frac{\int_0^{\infty} (t - U_1')^r h(t) dt}{\int_0^{\infty} h(t) dt} \quad (A 4)$$

where  $h(t)$  is the ordinate of the IUH at time  $t$ .

The standard formulae relating moments about the origin and about the centre of area are (Kendall and Stuart, 1974)

$$U_r' = \sum_{j=0}^r \binom{r}{j} U_{r-j}(U_1')^j \quad (A 5)$$

$$U_r = \sum_{j=0}^r \binom{r}{j} U'_{r-j} (-U'_1)^j \quad (A 6)$$

From eqns.(A 5) and (A 6), the relationships between the first four moments about the centre of area and the origin may be expressed by

$$\begin{aligned} U_2 &= U_2 + (U'_1)^2 \\ U_3 &= U_3 + 3U'_1 U_2 + (U'_1)^3 \\ U_4 &= U_4 + 4U'_1 U_3 + 6(U'_1)^2 U_2 + (U'_1)^4 \end{aligned} \quad (A 7)$$

or inversely by

$$\begin{aligned} U_2 &= U_2 - (U'_1)^2 \\ U_3 &= U_3 - 3U'_1 U_2 + 2(U'_1)^3 \\ U_4 &= U_4 - 4U'_1 U_3 + 6(U'_1)^2 U_2 - 3(U'_1)^4 \end{aligned} \quad (A 8)$$

#### COMPARISON OF IUH SHAPES

Nash (1960) proposed a procedure for scale-free comparison of the shapes of instantaneous unit hydrographs in which the second and the higher moments about the centre of area were expressed as dimensionless moments (shape factors) by division by the appropriate power of the first moment about the origin  $U'_1$ . The  $r$ -th dimensionless central moment  $m_r$  was defined by

$$m_r = \frac{U_r}{(U'_1)^r} \quad (A 9)$$

Different instantaneous unit hydrographs were represented on a shape factor diagram obtained by plotting the third dimensionless moment  $m_3$  as a function of the second dimensionless moment  $m_2$

$$m_3 = f(m_2) \quad (A 10)$$

A one-parameter instantaneous unit hydrograph would be represented by a single point on such a diagram while a two-parameter IUH would be represented by a line segment and one of three or more parameters by a region or a number of line segments. Instantaneous unit hydrographs of similar shapes would be represented approximately by closely related points or line segments or regions of the diagram. The flexibility, or generality, of a particular instantaneous unit hydrograph equation might be judged by the extent of the corresponding region of its  $m_3:m_2$  relationship, the smaller the region the less general and the more restrictive the equation. Obviously, other similar shape factor diagrams could be obtained by plotting higher dimensionless moments against the second dimensionless moment  $m_2$  and these could also be considered in such comparison of shapes.

#### THE MOMENTS OF THE INSTANTANEOUS UNIT HYDROGRAPH OF A CASCADE OF UNEQUAL LINEAR RESERVOIRS

For a cascade of  $n$ , not necessarily equal, linear reservoirs, the first moment about the origin, and the second and

third moments about the mean or centre of area of the corresponding IUH are given by

$$U'_1 = \sum_{i=1}^n k_i \quad (A 11)$$

$$U_2 = \sum_{i=1}^n k_i^2 \quad (A 12)$$

$$U_3 = 2 \sum_{i=1}^n k_i^3 \quad (A 13)$$

where  $k_i$  is a positive quantity of time representing the storage coefficient used in defining the storage discharge relationship ( $s_i = k_i q_i$ ) of the  $i$ -th reservoir in the cascade. For such a cascade, the relationship between the third dimensionless moment  $m_3$  and the second dimensionless moment  $m_2$  can be expressed by the following inequality (Nash, 1960);

$$2m_2^2 \leq m_3 \leq 2m_2^{1.5} \quad (A 14)$$

with

$$0 \leq m_3 \leq 2 \text{ and } 0 \leq m_2 \leq 1$$

Similarly, it can be shown algebraically that the relationship between the fourth dimensionless moment  $m_4$  and the second dimensionless moment  $m_2$  is given by the following inequality

$$3m_2^2 + 6m_2^3 \leq m_4 \leq 9m_2^2 \quad (A 15)$$

with

$$0 \leq m_4 \leq 9 \text{ and } 0 \leq m_2 \leq 1$$

The relationship of (A 14) corresponds to the loop in Fig. 4a. The corresponding relationship of (A 15) is shown in Fig. 4b.

In both figures, the lower boundaries ( $m_3 = 2m_2^2$  in Fig. 1a and  $m_4 = 3m_2^2 + 6m_2^3$  in Fig. 1b) correspond to a cascade of equal linear reservoirs. The corresponding IUH has an equation of gamma distribution form

$$h(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} e^{-\left(\frac{t}{k}\right)}$$

The upper boundaries ( $m_3 = 2m_2^{1.5}$  in Fig. 4a and  $m_4 = 9m_2^2$  in Fig. 4b) correspond to a lag and route IUH (lagging by an amount  $T$  and routing through a single reservoir having storage coefficient  $k$ ) whose equation is

$$\begin{aligned} h(t) &= 0 & t < T \\ &= \frac{1}{k} e^{-\left(\frac{t-T}{k}\right)} & t \geq T \end{aligned} \quad (A 16)$$

Any cascade of linear reservoirs, without restriction on relative sizes, with inflow at the upper end only, and discharge at the lower end of the cascade, will have an IUH represented by a single point within each of the loops of

Figs. 4a and 4b. Further restrictions were given by Nash (1960) for particular numbers of reservoirs in the cascade.

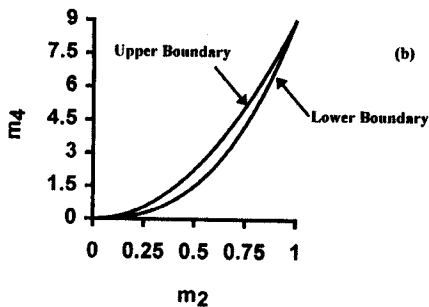
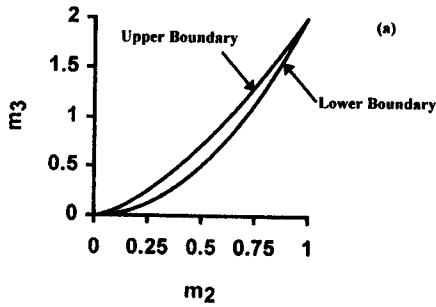


Fig. 4. Shape factor diagrams.

RESULTS OF THE SIMULATION EXPERIMENTS

Knowing the contributing areas and the storage coefficient values for a particular simulation, the first moment about the origin and the second and third moments about the centre of area of the IUH of each cascade are found using eqns.(A 11) to (A 13), respectively. Next, the first three moments about the origin are found using eqn.(A 7) and,

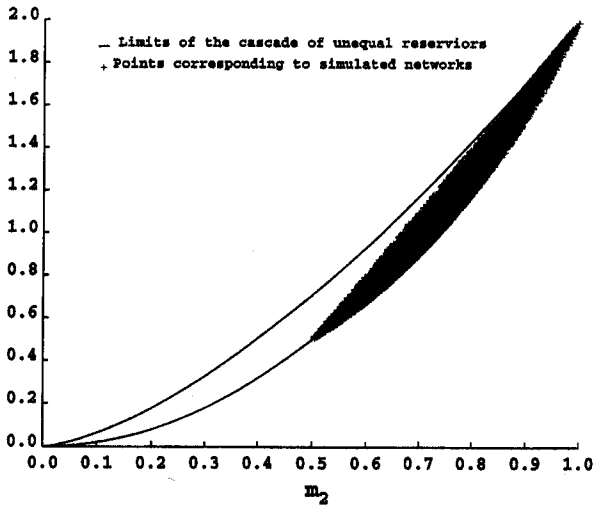


Fig. 5. The  $m_3:m_2$  diagram for simulated second level networks.

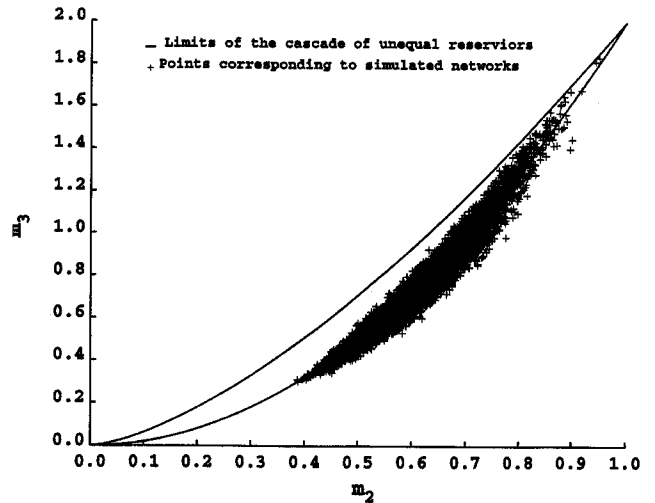


Fig. 6. The  $m_3:m_2$  diagram for simulated third level networks.

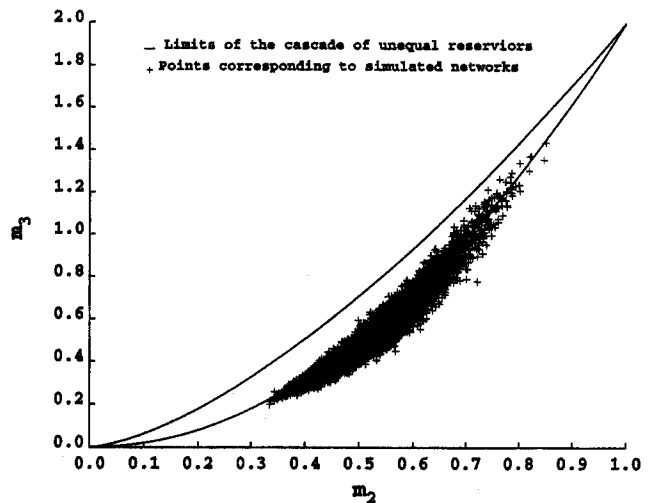


Fig. 7. The  $m_3:m_2$  diagram for simulated fourth level networks.

finally, the moments of the IUH of the network about the origin and centre of area are found utilizing eqns (A 2) and (A 8).

For a given network level, the  $m_3:m_2$  relationship was examined for ten thousand different simulations of possible network configurations. Figs. 5 to 9 show the relationships for random networks of levels two to six. Inspection shows that, for each of the chosen levels, the  $m_3:m_2$  relationship extends over a very limited region of the plane, close to the line corresponding to the gamma distribution model. It is clear that the extra complexity of the more general network is not reflected in a significantly greater generality of IUH shape than that of the simpler gamma distribution model.

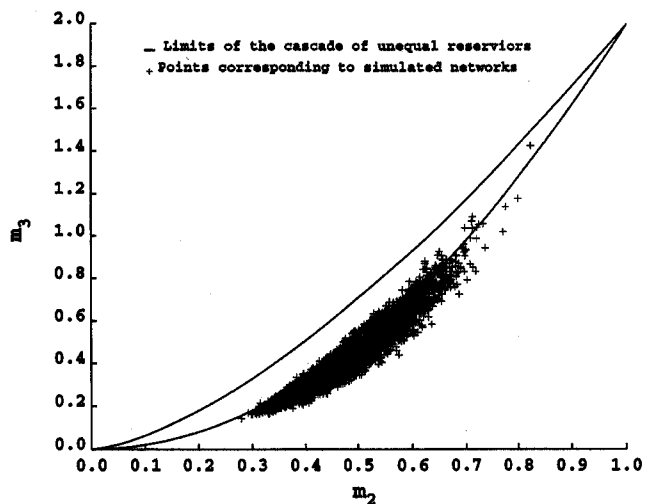


Fig. 8. The  $m_3:m_2$  diagram for simulated fifth level networks.

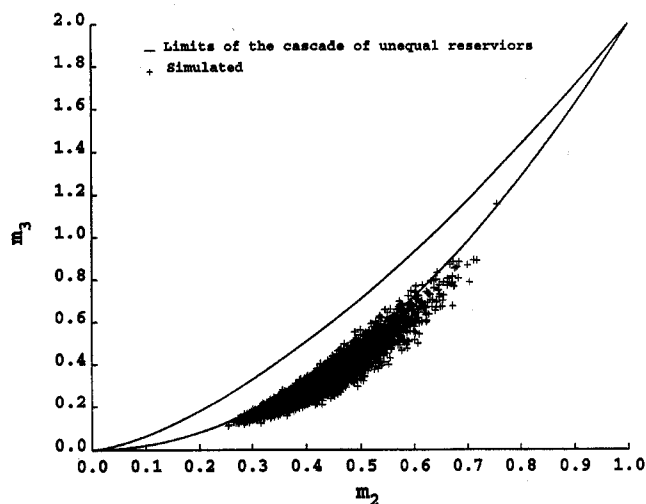


Fig. 9. The  $m_3:m_2$  diagram for sixth level networks.

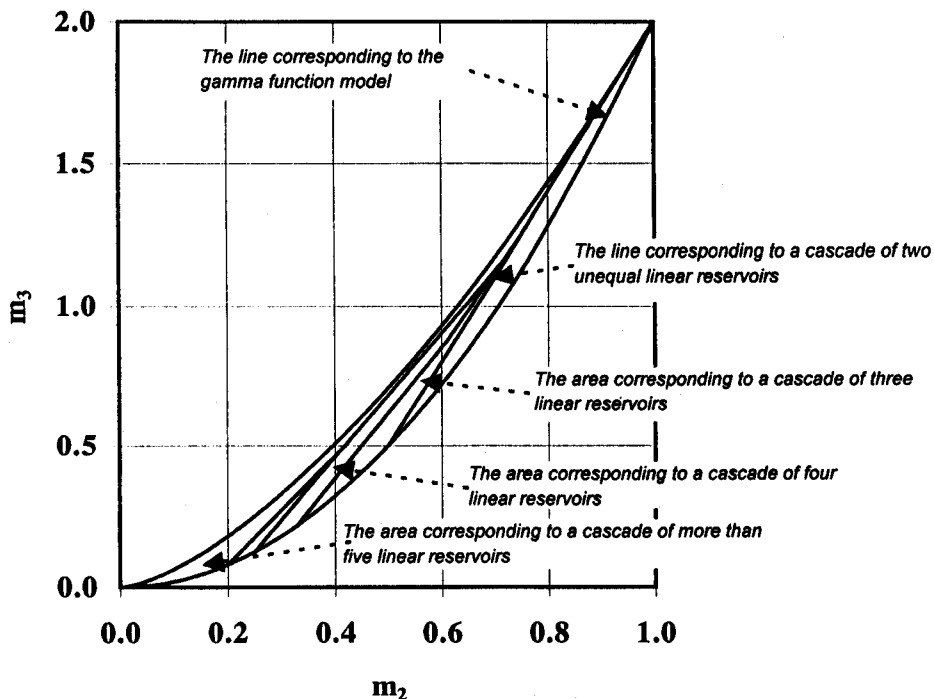


Fig. 10.  $m_3:m_2$  diagram for different numbers of linear reservoirs in the cascade after Nash (1960).

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