



# Supplement of

# A conceptual, distributed snow redistribution model

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### 1 Monte Carlo simulations to investigate equifinality issues

5 In this supplement we describe a Monte Carlo approach to investigate equifinality issues on 6 the snow transport model implemented in the hydrological model COSERO (Nachtnebel et 7 al., 1993). The snow module of the hydrological model COSERO uses 15 parameters. 8 Considering lateral snow transport adds two more parameters, namely  $H_v$  for snow holding 9 capacity due to vegetation and roughness of the terrain and C for adjusting snow transport. 10 The majority of the parameters however can be estimated a priori on the basis of literature or expertise of the modeller. Nevertheless, equifinality remains an issue, as it does in every 11 12 model that uses parameters that need calibration.

### 13 **1.1 Description of the Monte Carlo approach**

14 In a Monte Carlo simulation of 20000 runs, six parameters that are difficult to be estimated a 15 priori were varied within their meaningful boundaries. These parameters were D<sub>L</sub> and D<sub>U</sub>, T<sub>PR</sub> 16 and T<sub>PS</sub>, N<sub>VAR</sub> and C. D<sub>L</sub> and D<sub>U</sub> refer to the respective lower and upper boundaries of the 17 snow melt factor.  $T_{PR}$  and  $T_{PS}$  control the temperature range where liquid and solid 18 precipitation occur simultaneously. At and above temperature  $T_{RP}$  precipitation is pure liquid, 19 at and below T<sub>PS</sub> precipitation is pure solid. In between those two boundaries, the proportion 20 of solid to liquid precipitation is estimated linearly. N<sub>VAR</sub> determines the standard deviation of 21 the log-normal distribution that is used to describe sub-grid variability of snow depths within 22 a grid cell and C is a correction coefficient for adjusting the transport rate to the adjacent grid cell(s). 23

Instead of generating random values for each parameter in each grid cell, random delta values have been generated. Those apply to parameters in every cell that have been found during the calibration procedure using Rosenbrock's automated optimization routine (Rosenbrock, 1960). The spatial parameter distribution is based on process based assumptions. For instance, values for  $D_U$  and  $D_L$  depend on the elevation, slope and the land-use of a grid cell. The minimum and maximum values found by this are given in Table 1. Applying only delta values to the parameters has the advantage that the spatial relationship of the distributed parameters can be preserved. For instance, higher values for N<sub>VAR</sub> are assigned to grid cells that have a
high vertical gradient than for flat grid cells.

Some constraints were considered for generating random parameter sets: (1) In none of the 3 4 grid cell  $D_L$  can drop below zero and (2)  $D_U$  always needs to be higher than  $D_L$ . (3) The maximum valid value of  $D_U$  is assumed to be 10 mm °C<sup>-1</sup> d<sup>-1</sup>. (4) T<sub>PR</sub> needs to be higher than 5 6 T<sub>PS</sub> and (5) T<sub>PR</sub> cannot be below 0 °C or above 4 °C. (6) N<sub>VAR</sub> cannot drop below 0 and 7 cannot exceed 2.5 and (7) no values below 0 or above 2 are allowed for C. If a parameter set 8 did not fulfil these constraints it was rejected and a new parameter set was generated. Each 9 parameter set was used to run both model A and B. In this supplement, model A refers to the 10 model accounting for lateral snow transport while B refers to the standard model approach.

#### 11 **1.2 Results and discussion**

The results of model A and B using the parameter sets derived from the Monte Carlo simulations are shown in Fig. 1. The x-axis of a) refers to the Kling-Gupta-Efficiency regarding discharge and b) shows the behaviour of the models with respect to snow accumulation. While both models perform similarly well regarding runoff model B generates "snow towers" of up to 2400 mm SWE by the end of the modelled time series. The vast majority of realizations of model A show accumulations equal or less than 500 mm SWE.

18 In Fig. 2 the generated delta values of all varied parameters are plotted against the model 19 efficiency regarding discharge. The parameter  $D_U$  (Fig. 2 a, b) clearly is the most sensitive 20 parameter, followed by D<sub>L</sub> (Fig. 2 c, d) and T<sub>PS</sub> (Fig. 2 e, f). No clear conclusions can be made 21 from the other parameters. Due to accounting for snow transport to lower grid cells model A 22 is able to compensate for low D<sub>U</sub> values. Model B does not have this ability and consequently 23 the best results are achieved using values of  $D_U$  that are higher than the optimized  $D_U$  values 24 of model A. Since D<sub>L</sub> is most important in the accumulation season it has less influence on the 25 behaviour of both models. Interestingly both model A and B perform better the lower the 26 value of D<sub>L</sub> and the higher the value of T<sub>PS</sub>. Consequently both models perform best if the 27 amount of snow during the accumulation season is high.

The red triangles in Fig. 2 refer to the parameter sets found by the calibration using Rosenbrock's optimization routine. One has to keep in mind that this routine searches for a local optimum. Beginning with a parameter set well suited for the use in model A it might not find the globally best parameter set for model B and vice versa. This shows the limitations of

1 a local optimization function. For further work, the use of a global optimization function 2 should be considered. One has to keep in mind however that the optimal parameter set cannot be determined by Fig. 2. A six-dimensional matrix would be needed for that. 3

- 4 Kling et al., (2006) derived values for day degree rates for Austria from the mean radiation 5 index, the aspect, slope and elevation on a 1 x 1 km raster. They reported a range for  $D_{L}$  of 1.2 to 2.2 and for  $D_{\rm II}$  for 2.0 to 3.0 mm °C<sup>-1</sup> d<sup>-1</sup>. These values might be interpreted as physically 6 7 derived and therefore considered as realistic values for day degree parameter values. Most 8 modellers, however, would tend to use higher values at least for D<sub>U</sub>. Model A allows the 9 modeller to use D<sub>U</sub> values within or close to the range proposed by Kling et al., (2006), while 10 model B lead to the best results if higher and therefore unrealistic D<sub>U</sub> values are used.
- Consequently, model A allows for the use of more realistic boundaries of the snow melt 11
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factors than model B does.

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 $N_{VAR} \\$ С  $D_{\mathrm{L}}$  $D_{U} \\$  $T_{PR}$  $T_{PS} \\$ Minimum 0.0 0.27 1.11 -3.25 0.01 0.2 Maximum 1.45 5.79 1.37 -2.8 1.54 1.2

Table 1: Minimum and maximum values of the parameters found by the calibration using a
Rosenbrock's automated optimization routine.

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Figure 1. Performance of model A and B regarding discharge (a) and snow accumulation at the end of the modelled time series (b). While both model A and B perform similar with respect to runoff, in most of the model realizations of model A no extensive accumulation of snow can be observed whereas model B leads to snow accumulations of up to 2400 mm SWE.



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Figure 2: Sensitivity of the varied parameters. Model A is able to compensate for low values of the snow melt factor (a) while model B is not (b). The other parameters tested in this study seem to have less an effect on the model efficiency. The red triangles refer to the parameter set found by the calibration using Rosenbrock's optimization routine.