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# Building long-term and high spatio-temporal resolution precipitation and air temperature reanalyses by mixing local observations and global atmospheric reanalyses: the ANATEM model

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## Additional specifications on the ANATEM formulation for precipitation reconstruction (see section 3.3.2 of the paper)

As described in the Sect. 3.3.2 of the paper and shown in Fig. 4, both a simple additive and a simple multiplicative formulation of the correction are likely to produce anomalous values of the reconstructed precipitation.

The two panels of Fig. 4 show that the multiplicative formulation is suitable for low values of the precipitation estimate $\widehat{P}_{\mathrm{LM}}(d)$ obtained with the local model (LM) for the target day $d$ while the additive formulation is suitable for high values of $\widehat{P}_{\mathrm{LM}}(d)$.

The idea of the proposed formulation – inspired by the work of Dufour and Garçon (1997) for the assimilation of streamflow data in a hydrological model – is therefore to obtain a multiplicative behaviour for low values of $\widehat{P}_{\mathrm{LM}}(d)$ and an additive behaviour for high values of $\widehat{P}_{\mathrm{LM}}(d)$.

The proposed analytical formulation has these characteristics, as shown by its asymptotic behaviour when $x$ tends to zero or infinity, obtained through a Taylor expansion and detailed below.

$$f(x) = \frac{x^2 + a \cdot x}{x + b} \tag{1}$$

$f(x)$ can also be written :

$$f(x) = x \cdot \left(1 + \frac{a}{x}\right) \cdot \left(1 + \frac{b}{x}\right)^{-1} \tag{2}$$

Using the usual first order Taylor expansion $(1 + y)^{-1} = 1 + y + o(y)$ when $y$ is close to 0 for the variable $y = \frac{b}{x}$ :

$$x \cdot \left(1 + \frac{a}{x}\right) \cdot \left(1 + \frac{b}{x}\right)^{-1} \underset{x \to \infty}{\sim} x \cdot \left(1 + \frac{a}{x}\right) \cdot \left(1 - \frac{b}{x}\right) \tag{3}$$

$$\underset{x \to \infty}{\sim} x + a - b + \frac{a \cdot b}{x} \tag{4}$$

As the last term $\frac{a \cdot b}{x}$ tends to 0 when $x$ tends to infinity,

$$f(x) = x \cdot \left(1 + \frac{a}{x}\right) \cdot \left(1 + \frac{b}{x}\right)^{-1} \underset{x \to \infty}{\sim} x + a - b \tag{5}$$

When $x$ tends to 0, the following limit is obtained for $f(x)$:

$$f(x) = \frac{x^2 + a \cdot x}{x + b} = x \cdot \frac{x + a}{x + b} \underset{x \to 0}{\sim} x \cdot \frac{a}{b} \tag{6}$$

In the case of the ANATEM formulation, $f(x)$ is written $\widehat{P}^k_{\mathrm{ANATEM}}(d)$, the variable is $x(d) = \widehat{P}_{\mathrm{LM}}(d)$, and the two parameters $a$ and $b$ have to be expressed as a function of $P(d_k)$ and $\widehat{P}_{LM}(d_k)$.

$$\widehat{P}^k_{\mathrm{ANATEM}}(d) = f(x(d)) = \frac{x(d)^2 + a(d_k) \cdot x(d)}{x(d) + b(d_k)} \tag{7}$$

As explained in the paper, the two following conditions have been set to define the parameters:

- The slope of the tangent to the curve at $x = 0$ must be $\left(\frac{P(d_k)}{\widehat{P}_{LM}(d_k)}\right)^2$ ;
- When $P(d_k) = \widehat{P}_{LM}(d_k)$, the equality $\widehat{P}^k_{\mathrm{ANATEM}}(d) = \widehat{P}_{\mathrm{LM}}(d)$ must be obtained.

The first condition has been imposed empirically and selected because it gave satisfactory results both for streamflow data assimilation and in the case of the ANATEM model, while the second condition is logically deduced from the idea of the correction model.

The first condition means that the derivative of the function $f$ around 0 must be $\left(\frac{P(d_k)}{\widehat{P}_{LM}(d_k)}\right)^2$. The derivative of the function $f$ is:

$$f'(x) = \frac{x^2 + 2b \cdot x + a \cdot b}{(x + b)^2} \tag{8}$$

so $f'(0) = \frac{a}{b}$ and the first condition gives:

$$\frac{a}{b} = \left(\frac{P(d_k)}{\widehat{P}_{LM}(d_k)}\right)^2 \tag{9}$$

According to the second condition, if $P(d_k) = \widehat{P}_{LM}(d_k)$ then $f(x) = x$, which means that $a = b$

We finally have the two following relationships:

$$\frac{a}{b} = \left(\frac{P(d_k)}{\widehat{P}_{LM}(d_k)}\right)^2 \tag{10}$$

$$P(d_k) = \widehat{P}_{LM}(d_k) \Leftrightarrow a = b \tag{11}$$

which give the following parameters:

$$a = P(d_k) \tag{12}$$

$$b = \frac{\widehat{P}_{LM}(d_k)^2}{P(d_k)}. \tag{13}$$

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## References

Dufour, C. and Garçon, R.: Méthode statistique de recalage du modèle de prévision au pas journalier MORDOR dans le cadre du projet Vienne, Rapport technique, Electricité de France, Grenoble, France, 1997.