Comparison of the results by the present method and the fourth order Runge-Kutta method

For comparing the result obtained by the present method and that by the fourth order Runge-Kutta method, we take the Example 5.8 in the book of K. Subramanya (2009). The book title is “Flow In Open Channels.” The followings show: (1) the content of Example 5.8, (2) the numerical code **for Matlab** by using the present method (Jan & Chen’s method), (3) the numerical code for Matlab by using the standard fourth order Runge-Kutta method, and the comparison of the results by the present method and fourth order Runge-Kutta method.

Example 5.8

A river 100 *m* wide and 3.0 *m* deep has an average bed slope of 0.0005. Estimate the length of GVF profile produced by a low dam which raises the water surface just upstream of it by 1.50 *m*. Assume the channel cross-section is rectangular, the Manning coefficient n equals 0.035, and the normal depth of flow is 3.0 *m*. Find the GVF profile (M1 curve).

Numerical code for Matlab by the present method (Jan & Chen’s method)：

```
clear all;clc;tic
%-------Basic Information--------
h0=3;                      % Normal depth
S0=0.0005;                 % Average bed slope
n=0.035;                   % Manning coefficient
h1=4.5;                    % Water depth (m) at boundary condition, x＝0
g=9.81;                    % Gravitational acceleration (m/s^2)
q=h0^(5/3)*S0^(1/2)/n;     % Unit-width discharge for wide channels by Manning equation (m^3/s/m)
hc=(q^2/g)^(1/3);          % Critical depth (m)
N=10/3;                    % Hydraulic exponent for uniform-flow computation
M=3;                       % Hydraulic exponent for critical-flow computation in wide channels
hEND=3.03;                 % Upstream water depth (m)
xs=0;                      % Boundary condition at which the start of GVF calculation
u1=h1/h0;                  % Dimensionless water depth at downstream BC x＝0
XS=xs*S0/h0;               % Dimensionless longitudinal coordinate
ld=hc/h0;                  % Dimensionless critical depth
a=1;                       % Parameter used in Gaussian hypergeometric function
b1=-1/N;                   % Parameter used in Gaussian hypergeometric function
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```
c1=b1+1;                    % Parameter used in Gaussian hypergeometric function
b2=(M-1)/N;                 % Parameter used in Gaussian hypergeometric function
c2=b2+1;                    % Parameter used in Gaussian hypergeometric function
z1=u1^(-N);                 % Dimensionless water-depth parameter used in GHF
hypergeomBF1= hypergeom([a,b1], c1, z1)     % GHF-1 at BC
hypergeomBF2= hypergeom([a,b2], c2, z1)     % GHF-2 at BC

%------------M1 Curve calculated by Jan &Chen's method --------------------
  for i=1:100000                % Upper-limit of calculation steps
      h(i)=h1-0.01*(i-1);      % Water depth (m) at a specified longitudinal location to be determined
      u(i)=h(i)/h0;             % Dimensionless variable of water depth
      z(i)=u(i)^(-N);           % Dimensionless variable used in GHF
      if y(i)<hEND              % Condition for the end of GVF calculation
         break
      end
Xx0(i)=XS+(u(i)*hypergeom([a,b1], c1, z(i))-u1*hypergeomBF1)+
ld^M*(u(i)^(-M+1)*hypergeom([a,b2], c2, z(i))-u1^(-M+1)*hypergeomBF2)/(M-1);
         Xx(i)=Xx0(i)*h0/(S0);          % transfer the longitudinal coordinate to be dimensional
      if h(i)==hEND                     % Condition for the end of GVF calculation
         break
      end
  end
      h=h(1:length(Xx));                % Let storage length for water depth equal that for distance Xx
toc
```

Numerical code for Matlab by the standard fourth order Runge-Kutta method：

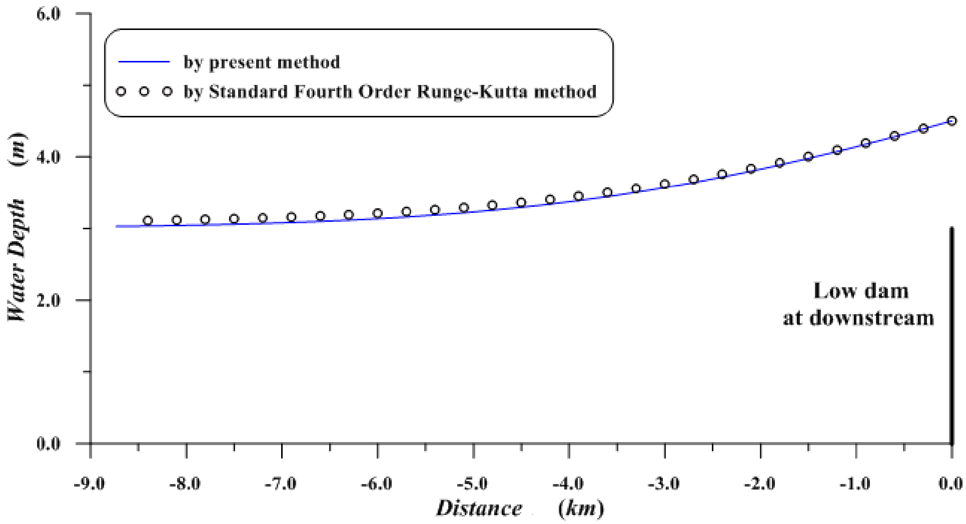
```
clear all;clc;format long;tic
%-------Basic    Information--------
h0=3;                        % Normal depth (m)
S0=0.0005;                   % Average bed slope
n=0.035;                     % Manning coefficient
h1=4.5;                      % Water depth (m) boundary condition：x＝0
g=9.81;                      % Gravitational acceleration (m/s^2)
q=h0^(5/3)*S0^(1/2)/n;       % Unit-width discharge for wide channels by Manning equation (m^3/s/m)
hc=(q^2/g)^(1/3);            % Critical depth (m)
B=100;                       % Width of a rectangular channel (m)
```

```
Q=q*B;                          % Flow discharge in a rectangular channel (m3/s)
T=B;                            % Top width of a rectangular channel (m)
dx=-1;                          % Distance interval (m)
L=-8644;                        % Computation end of longitudinal distance (m)
nn=L/dx;                        % Number of calculation steps
h=[];                           % Water depth (m) for the longitudinal datum
%----------Calculation by Standard Fourth Order Runge-Kutta method solved by Matlab ------------
for i=1:nn
     LL(i)=dx*i;                              % Storage for longitudinal distance length
%-------K1--------------
    A1=B*h1;                                  % Cross-section area of flow in a rectangular channel (m2)
    P1=2*h1+B;                                % Wetted perimeter of flow in a rectangular channel (m)
    R1=A1/P1;                                 % Hydrau1lic radius (m)
    V1=Q/A1;                                  % Average velocity (m/s)
    Sf1=(V1^2)*(n^2)/(R1^(4/3));              % Energy slope evaluated by Manning equation
    F1=(S0-Sf1)/(1-Q^2*T/(g*A1^3));           % Function in the basic differential equation of GVF
    K1=dx*F1;                                 % First parameter (m)
%-------K2---------------
    A2=B*(h1+K1/2);                           % Cross-section area of flow in a rectangular channel (m2)
    P2=2*(h1+K1/2)+B;                         % Wetted perimeter of flow in a rectangular channel (m)
    R2=A2/P2;                                 % Hydrau1lic radius (m)
    V2=Q/A2;                                  % Average velocity (m/s)
    Sf2=(V2^2)*(n^2)/(R2^(4/3));              % Energy slope evaluated by Manning equation
    F2=(S0-Sf2)/(1-Q^2*T/(g*A2^3));           % Function in the basic differential equation of GVF
    K2=dx*F2;                                 % Second parameter (m)
%------K3---------------
    A3=B*(h1+K2/2);                           % Cross-section area of flow in a rectangular channel (m2)
    P3=2*(h1+K2/2)+B;                         % Wetted perimeter of flow in a rectangular channel (m)
    R3=A3/P3;                                 % Hydrau1lic radius (m)
    V3=Q/A3;                                  % Average velocity (m/s)
    Sf3=(V3^2)*(n^2)/(R2^(4/3));              % Energy slope evaluated by Manning equation
    F3=(S0-Sf3)/(1-Q^2*T/(g*A3^3));           % Function in the basic differential equation of GVF
    K3=dx*F3;                                 % Third parameter (m)
%-----K4--------------
    A4=B*(h1+K3/2);                           % Cross-section area of flow in a rectangular channel (m2)
    P4=2*(h1+K3/2)+B;                         % Wetted perimeter of flow in a rectangular channel (m)
    R4=A4/P4;                                 % Hydrau1lic radius (m)
    V4=Q/A4;                                  % Average velocity (m/s)
```

```
    Sf4=(V4^2)*(n^2)/(R4^(4/3));          % Energy slope evaluated by Manning equation
    F4=(S0-Sf4)/(1-Q^2*T/(g*A4^3));       % Function in the basic differential equation of GVF
    K4=dx*F4;                             % Fourth parameter (m)

    h2=h1+(K1+2*K2+2*K3+K4)/6;            % Water depth (m) at the subsequent step
    h1=h2;                                % Renew water depth (m) for next-step calculation
    h=[h h2];                             % Storage length for water depth
end
toc
```

Comparison of the results by the present method and fourth order Runge-Kutta method



The numerical error in water depth obtained by the standard fourth order Runge-Kutta method is about 2% at the longitudinal distance x = -8 km.