



# A model based on dimensional analysis for prediction of nitrogen and phosphorus concentrations at the river station Ižkovce, Slovakia

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**Abstract.** The aim of this paper is to develop a model for pollutant concentration prediction in a stream. The developed model that determines nitrogen and phosphorus concentrations in a river is based on a dimensional analysis. Application of dimensional analysis to water quality modelling is presented, pointing out possibilities of applying this methodology in water quality research. We investigate how dimensional analysis can be applied to water quality modelling and which benefits it can bring to researchers in this area. For modelling water quality in a water stream it is essential to know the parameters that influence water quality. The relevant parameters are flow of water in the river (discharge), its catchment area, velocity of water in the stream, temperature of water, temperature of air and measured concentrations of the pollutant – nitrogen and phosphorus. A sensitivity analysis shows that the concentration of pollutant in water stream is sensitive to changes in both water and air temperatures. The model performs well when average values are used; the prediction error increases when the single concentration values are considered. The model was developed, calibrated and evaluated using measured data from the river station Ižkovce, River Laborec in eastern Slovakia.

aquatic systems (Wade et al., 2002b). The Water Framework Directive (2000/60) demands new approaches for managing and improving surface and groundwater quality across the European Union, with emphasis shifting from chemical towards ecological water quality standards. There are also older directives aimed at reducing the impact of nitrogen and phosphorus pollution, which include the Nitrate, Habitat and Waste-water Treatment Directives. Since many EU policies aim to control nutrients in river systems, there is a need to understand the combined result of their implementation. The recent emphasis on integrated catchment planning and unifying EU directives has led to considerable research interest in the study of river systems. In particular, the pursuit of models and modelling frameworks capable of predicting water quality across ranges of spatial and temporal scales has provided key motivation. There are many studies focusing on nutrient loads in rivers (Beaujouan et al., 2001; Butturini and Sabater, 2002; Cosby et al., 1997; Goolsby et al., 2000; Jarvie et al., 2002a,b; Johnes, 1996; Neal et al., 2006; Pieterse et al., 2003; Le et al., 2010; Wade et al., 2001; Wang and Lewis, 2009; Ruiz et al., 2002a, b). Several investigations have examined trends in nitrogen and phosphorus concentrations. Certain decreases in nutrient concentrations have been found in recent years in Swedish agricultural rivers (Ulén and Fölstrer, 2007), Latvian rivers (Stålnacke et al., 2003) and Slovakian rivers (Bendíková, 2004). This is most evident in areas where agricultural activities and use of fertilisers has decreased. Mainly human activities have increased the loads

## 1 Introduction

In recognition of the adverse impacts of pollutants and the need for integrated water management, the EU has introduced a series of directives aimed at reducing nutrients in

of total nitrogen and total phosphorus in running waters, and this may have a serious impact on their ecological quality.

European management strategies have tended to address single issues (e.g. diffuse or point sources) or particular regions (e.g. upland or lowland areas). However, it is recognised that the nutrient status of river systems reflects the combined contribution of sources: fertiliser inputs, atmospheric deposition and sewage discharges (Wade et al., 2002a). Superimposed on these anthropogenic inputs, an integrated management approach is required (Langan et al., 1997). In particular, such an approach is needed to assess the likely impacts of land management and climatic change on EU river nutrient concentrations and loads (Wade et al., 2002a). The importance of this is acknowledged in the United States of America, where an integrated approach is being implemented through permit trading between diffuse and point source polluters (EPA, 1999). There is no clear-cut distinction between point sources or diffuse sources (non-point sources) (Chapman, 1996). The major point sources of pollution to fresh waters originate from the collection and discharge of domestic waste waters, industrial wastes or certain agricultural activities, such as animal husbandry. Most other agricultural activities, such as pesticide spraying or fertiliser application, are considered as diffuse sources. In general, with increasing discharge point sources of contaminants are diluted, whereas diffuse sources show increased concentrations. In order to predict the pollutant concentrations in rivers it is necessary to model water quality. There are various mathematical models for water resource systems, as mentioned for example in Straškraba (1994); Rauch et al. (1998); Somlyódy et al. (1998); or Borah and Bera (2003).

During the past decades several models which predict the concentration profiles after a discharge of pollutants in a river have been developed. Application of these models to a river leads to discrepancies between predicted and measured concentration profiles.

Dimensional analysis is a well-known methodology in physics, chemistry and other traditional engineering areas. In its simplest form, dimensional analysis is used to check the meaningfulness of a set of equations (dimensional homogeneity). In the last century dimensional theory has been profoundly investigated: its highest achievement is the Buckingham theorem (or pi-theorem), which states that any equation modelling a physical problem can be rearranged in terms of non-dimensional ratios, thus limiting the variables to be handled, and especially enriching the inner physical knowledge of the studied phenomenon (Miragliotta, 2010).

We developed a model that predicts nitrogen and phosphorus concentrations in a river. The developed model that determines concentrations of pollutants in a water stream is based on dimensional analysis. The fundamentals of modelling pollutant prediction in a water stream consist in derivation of function dependency from expressed non-dimensional arguments. Non-dimensional arguments are stated from variables which influence the occurrence of pollutants. From this

function dependency it is possible to obtain concentration values of a pollutant in a water stream. In general this dependency has power law status. A model for prediction of nitrogen and phosphorus concentrations in a water stream has been developed for the river station Ižkovce, River Laborec in eastern Slovakia. The differences between the concentrations calculated from the developed model and measured concentrations are also discussed in this paper. The use of dimensional analysis for water quality modelling is a new approach here.

Sensitivity analysis (SA) is the study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input (Saltelli, 2002). Sensitivity and uncertainty analysis are important tools for exploring complex models. Saltelli et al. (2000), Kleijnen (1997) and Oakley and O'Hagan (2004) clearly show the key role of these tools within the wider context of the building, validation and use of process models.

## 2 Material and methods

The model describing pollutant concentration in a water stream is based on the formation of non-dimensional arguments  $\pi_i$  from the stated variables influencing the pollutant concentration. Their valuable feature is that in all existing systems of units they have the same numerical size and they have no dimension. Formation of the model consists in derivation of functional dependence from the expressed non-dimensional variables, which in general always has power law character. Transformation of this function into logarithmic coordinates gives it linear character, which makes working with the model easier and enables the parameters of the linear function to be determined (Čarnogurská, 2000; Čarnogurská et al., 2011; Mäsiar and Kamenský, 2001; Vilčeková and Šenitková, 2009).

The most important part for the model development is selection of appropriate variables. For determination of pollutant concentration in a water stream using dimensional analysis it is essential to state the parameters which characterise the water stream, and which may be measured (Zeleňáková and Švecová, 2006):

- flow  $Q$  [ $\text{m}^3 \text{s}^{-1}$ ], or mass flow  $Q_m$  [ $\text{kg s}^{-1}$ ],
- catchment area  $F$  [ $\text{m}^2$ ],
- velocity of water in the stream  $v$  [ $\text{m s}^{-1}$ ],
- temperature of water  $T_w$  [K],
- temperature of air  $T_a$  [K],
- pollutant concentration  $C$  [ $\text{kg m}^{-3}$ ].

All the given variables are presented in basic dimensions, which is the condition for dimensional analysis application.

The general relation among the selected variables, which can affect the pollutant concentration, can be put down in the next form in order that each parameter is considered with the same dimension  $\varphi(Q_m, F, \nu, T_w, C, T_a) = 0$ .

The dimensional matrix-relation (1) has the rank of matrix  $m = 4$  and its lines are dimensionally independent from each other. From  $n = 6$  independent variables at matrix rank  $m$ , it is possible to set up  $n - m$  of non-dimensional arguments.

$$\begin{matrix} & Q_m & F & \nu & T_w & C & T_a \\ \begin{matrix} m \\ s \\ \text{kg} \\ \text{K} \end{matrix} & \left| \begin{array}{cccccc} 0 & 2 & 1 & 0 & -3 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array} \right. & . & (1) \end{matrix}$$

The following equation is valid

$$\pi_i = Q_m^{x_1} \cdot F^{x_2} \cdot \nu^{x_3} \cdot T_w^{x_4} \cdot C^{x_5} \cdot T_a^{x_6}. \quad (2)$$

Matrix (1) is then changed into four linear equations with six unknown parameters.

The solution of matrix (1) consists of two independent vectors  $\pi_1$  and  $\pi_2$  containing various parameters in accordance with (3).

$$\begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \begin{matrix} \pi_1 \\ \pi_2 \end{matrix} & \begin{pmatrix} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1. \end{pmatrix} & & (3) \end{matrix}$$

Functional dependence of these arguments is expressed by Eq. (4). Another option for obtaining non-dimensional arguments  $\pi_1$  and  $\pi_2$  is a process resulting from the principle of dimensional analysis. One argument would be expressed directly as the ratio of the two simplex temperatures, which have the same dimension. The second argument would result from the solution of matrix (1), which would mean that from the two temperatures we would have only one – temperature of water  $T_w$ . The solution of such a system of linear equations would produce only one non-dimensional argument. This in turn would represent a functional relationship to the simplex expressed by Eq. (4).

The sought dimensional homogeneous function in non-dimensional form is  $\varphi(\pi_1, \pi_2) = 0$ . After adjustment is valid

$$\varphi\left(\frac{Q_m}{F \cdot \nu \cdot C}, \frac{T_a}{T_w}\right) = 0. \quad (4)$$

Non-dimensional argument  $\pi_1$  contains the unknown parameter  $C$  so this argument can be expressed as a function of argument  $\pi_2$  in the form

$$\pi_1 = \varphi(\pi_2). \quad (5)$$

The real course of the dependence (5) of non-dimensional arguments  $\pi_1$  to  $\pi_2$  is depicted in Figs. 2 and 3. The relation (5)

between independent argument  $\pi_2$  and dependent argument  $\pi_1$  can be defined by the following equation

$$\pi_1 = A \cdot \pi_2^B. \quad (6)$$

The regression coefficients can be calculated by the method of the least squares. The relation (6), characterizing the pollutant concentration in a river is obtained in the form

$$\frac{Q_m}{F \cdot \nu \cdot C} = A \cdot \left(\frac{T_a}{T_w}\right)^B. \quad (7)$$

After modification the following equation is valid

$$C = A^{-1} \cdot T_a^{-B} \cdot \nu^{-1} \cdot F^{-1} \cdot Q_m \cdot T_w^B. \quad (8)$$

Relation (8) represents the model of the pollutant concentration in water stream. The model is valid for each pollutant, but it is necessary to calculate new regression coefficients  $A$  and  $B$ .

Prediction of pollutant concentration in a water stream was performed in the River Laborec in eastern Slovakia – Fig. 1. Water quality in this river has been monitored in river stations over a long period.

Required data were obtained from the Slovakian Water Management Company in Košice and the Slovak Hydrometeorological Institute in Košice.

Data measured during the course of eight years, from 1995 to 2002 (12 values in a year) were used. Statistically processed data are presented in Table 1. There are 12 values representing the average values of each month over a period of 8 yr in. However, we used all the dataset of 96 values for each parameter for the model calibration.

It is possible to develop a model for prediction of the pollutant in each river station where the input parameters are known. This paper presents the development of the model for the prediction of nitrogen and phosphorus concentrations at the river station Laborec – Ižkovce, at river kilometre 10.30. The area of the catchment is 4468.067 km<sup>2</sup>.

The sensitivity analysis was performed evaluating the effect produced by the variation of the input data. Spider plots were used for the representation of the results following Eschenbach (1992), Oyarzun et al. (2007).

The model performance analysis included the determination of the root mean square error (RMSE), the relative RMSE (RRMSE) and the coefficient of residual mass (CRM), after the model adjustment for the time period considered. These indicators have the following expressions (Loague and Green, 1991; Antonopoulos, 2001):

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (C_{\text{predicted}} - C_{\text{measured}})^2}{n}} \quad (9)$$

$$\text{RRMSE} = \frac{\text{RMSE}}{C_{\text{average}}} \cdot 100 \quad (10)$$

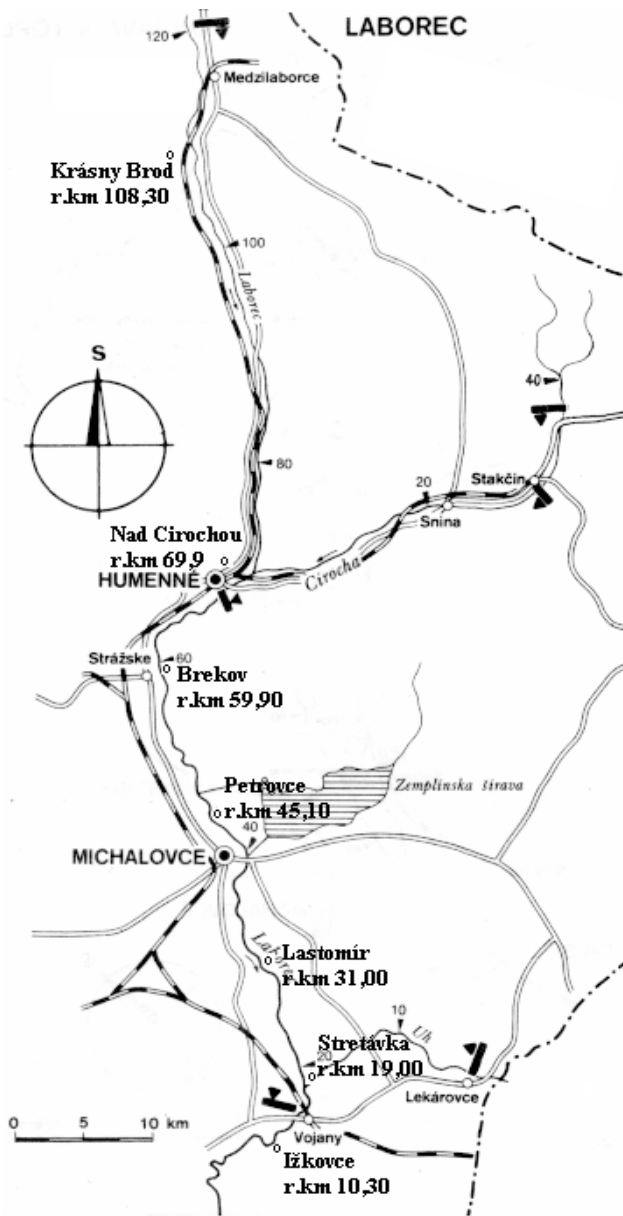


Fig. 1. River Laborec – eastern Slovakia with indication of the Ižkovce river station.

$$CRM = \frac{\left( \sum_{i=1}^n C_{\text{measured}} - \sum_{i=1}^n C_{\text{predicted}} \right)}{\sum_{i=1}^n C_{\text{measured}}} \quad (11)$$

where  $C_{\text{predicted}}$  and  $C_{\text{measured}}$  are calculated and measured values of pollutant concentration in river;  $C_{\text{average}}$  is mean of measured values,  $n$  is number of data.

The average uncertainty was calculated using the following equation

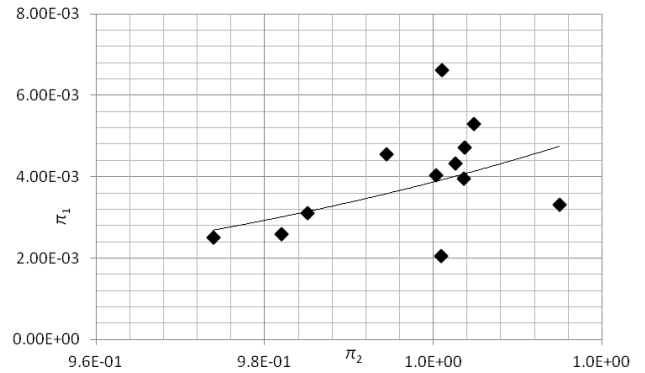


Fig. 2. Non-dimensional arguments and regression line for nitrogen concentrations.

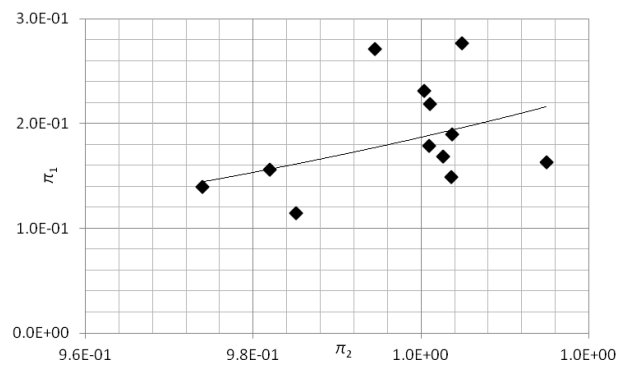


Fig. 3. Non-dimensional arguments and regression line for phosphorus concentrations.

$$\sigma = \frac{1}{n} \cdot \sum_{i=1}^n \frac{|C_{\text{measured}} - C_{\text{predicted}}|}{C_{\text{measured}}} \cdot 100. \quad (12)$$

### 3 Results and knowledge achieved

According to the known relevant parameters (from Table 1), the non-dimensional arguments were stated. Figure 2 depicts the dependency of non-dimensional arguments for nitrogen concentrations and Fig. 3 for phosphorus concentrations.

#### 3.1 Model calibration

The regression equation for nitrogen concentrations takes the form

$$\pi_1 = 0.0039\pi_2^{13.805} \quad (13)$$

and for phosphorus concentrations

$$\pi_1 = 0.1868\pi_2^{9.7892}. \quad (14)$$

The regression coefficients recalling Eqs. (13) and (14) are

–  $A = 0.0039$  (for nitrogen) and  $0.1868$  (for phosphorus),

**Table 1.** Values of relevant arguments and measured nitrogen and phosphorus concentrations.

| Month | $Q_m$<br>kg s <sup>-1</sup> | $v$<br>m s <sup>-1</sup> | $T_a$<br>K | $T_w$<br>K | $C_N$<br>kg m <sup>-3</sup> | $C_P$<br>kg m <sup>-3</sup> |
|-------|-----------------------------|--------------------------|------------|------------|-----------------------------|-----------------------------|
| 1     | 37343.80                    | 1.340                    | 272.60     | 277.60     | 0.002419                    | $4.00 \times 10^{-5}$       |
| 2     | 30409.00                    | 1.258                    | 270.34     | 277.58     | 0.002162                    | $3.88 \times 10^{-5}$       |
| 3     | 85402.00                    | 1.788                    | 278.04     | 279.58     | 0.002345                    | $3.94 \times 10^{-5}$       |
| 4     | 66219.60                    | 1.650                    | 281.20     | 281.10     | 0.002230                    | $3.88 \times 10^{-5}$       |
| 5     | 80227.20                    | 1.792                    | 290.56     | 289.16     | 0.001894                    | $3.62 \times 10^{-5}$       |
| 6     | 33690.60                    | 1.316                    | 295.90     | 291.54     | 0.001727                    | $3.52 \times 10^{-5}$       |
| 7     | 42386.00                    | 1.356                    | 295.06     | 294.78     | 0.003409                    | $3.92 \times 10^{-5}$       |
| 8     | 35370.60                    | 1.334                    | 297.58     | 296.80     | 0.001372                    | $3.52 \times 10^{-5}$       |
| 9     | 30224.80                    | 1.264                    | 293.78     | 292.72     | 0.001351                    | $3.60 \times 10^{-5}$       |
| 10    | 54306.20                    | 1.434                    | 287.96     | 287.66     | 0.001279                    | $3.88 \times 10^{-5}$       |
| 11    | 57952.25                    | 1.510                    | 285.03     | 283.98     | 0.001820                    | $4.52 \times 10^{-5}$       |
| 12    | 30954.40                    | 1.266                    | 276.00     | 280.18     | 0.001761                    | $4.78 \times 10^{-5}$       |

–  $B = 13.805$  (for nitrogen) and  $9.7892$  (for phosphorus).

Pollutant concentrations were calculated according to Eq. (8), the developed model for prediction of pollutant concentration in a water stream; on the basis of measured input parameters and determined regression coefficients.

In Tables 2 and 3 the values of measured and predicted nitrogen and phosphorus concentrations are compared.

### 3.2 Model verification

The model for the calculation of nitrogen and phosphorus concentrations was tested in the eight-year period: 2003–2010.

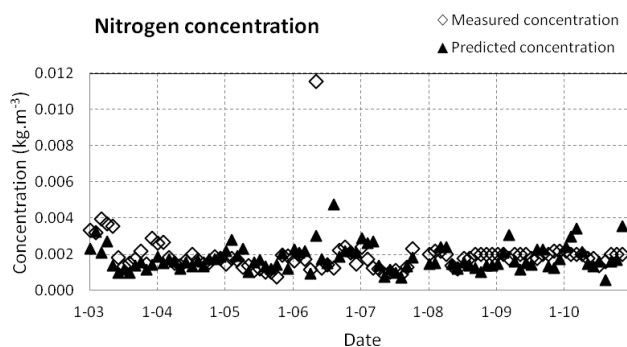
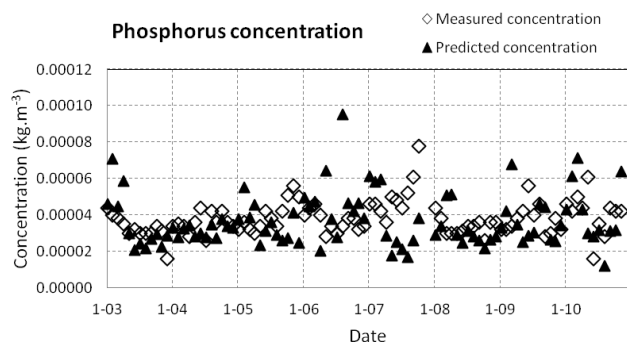
Comparison of measured and predicted nitrogen concentration in river is shown in the Fig. 4. Figure 5 depicts measured and predicted phosphorus concentration in River Laborec – Ižkovce river station.

There were 96 measured and predicted values of nitrogen and phosphorus concentrations, as shown in Figs. 4 and 5. The average uncertainty was calculated according to Eq. (12) and the result was 31.33 % for the verified nitrogen model and 32.30 % for the verified phosphorus model. This is appropriate for modelling such phenomena as concentration of pollutants in a water stream as it can be influenced by weather condition as well as human error (Zeleňáková and Čarnogurská, 2008). The proper variables for dimensional analysis also have to be used.

Model sensitivity to input data is described in following.

### 3.3 Model sensitivity

Figures 6 and 7 show results from sensitivity analysis how the model is sensitive to the choice of the parameters. The figures depict the effect on the pollutant concentration due to the different input data. The temperature of air and the temperature of water have the major influences on the nitrogen and phosphorus concentration. The model shows the highest sensitivity to the temperature of air. The nitrogen and

**Fig. 4.** Comparison of measured and predicted nitrogen concentrations in river.**Fig. 5.** Comparison of measured and predicted phosphorus concentrations in river.

phosphorus concentrations predicted by the model show little sensitivity to the flow and velocity of water in the stream. The catchment area has no effect on change of pollutant concentration so it was not included in the plots.

It should be stated that the model predicts higher values of pollutant concentrations in river than measured values ( $CRM < 0$ ). Predicted error (RRMSE) magnitude is 35 % for single values of nitrogen concentration and 38 % for phosphorus (Table 4). If only average measured and predicted values are considered, the magnitude of error diminishes (Table 5).

## 4 Discussion

The Buckingham  $\pi$  theorem is the core of the dimensional analysis. This theorem describes how every equation involving  $n$  variables can be rewritten as an equation of  $n - m$  dimensionless parameters, where  $m$  is the number of fundamental dimensions used. Furthermore, it provides a method for computing these non-dimensional arguments from the given variables. This provides a method for computing sets of these parameters from the given variables, even if the form of the equation is still unknown. The choice of non-dimensional arguments is not unique: Buckingham's theorem

**Table 2.** Values of non-dimensional arguments, predicted concentrations, uncertainty for nitrogen.

| Month | $C_{\text{measured}}$<br>kg m <sup>-3</sup> | $\pi_2$<br>– | $\pi_1$<br>– | $C_{\text{predicted}}$<br>kg m <sup>-3</sup> | $\sigma$<br>% |
|-------|---|--------------|--------------|--|---------------|
| 1     | 0.002419                                    | 0.981988     | 0.002052     | 0.002055                                     | 15.03         |
| 2     | 0.002162                                    | 0.973917     | 0.001670     | 0.001998                                     | 7.59          |
| 3     | 0.002345                                    | 0.994492     | 0.003341     | 0.002958                                     | 26.13         |
| 4     | 0.002230                                    | 1.000356     | 0.002705     | 0.002292                                     | 2.75          |
| 5     | 0.001894                                    | 1.004842     | 0.003431     | 0.002403                                     | 26.92         |
| 6     | 0.001727                                    | 1.014955     | 0.001885     | 0.001197                                     | 30.68         |
| 7     | 0.003409                                    | 1.000950     | 0.002610     | 0.001770                                     | 48.06         |
| 8     | 0.001372                                    | 1.002628     | 0.002061     | 0.001467                                     | 6.92          |
| 9     | 0.001351                                    | 1.003621     | 0.001939     | 0.001305                                     | 3.36          |
| 10    | 0.001279                                    | 1.001043     | 0.002903     | 0.002142                                     | 67.44         |
| 11    | 0.001820                                    | 1.003698     | 0.003123     | 0.002093                                     | 15.03         |
| 12    | 0.001761                                    | 0.985081     | 0.001754     | 0.001727                                     | 1.96          |

**Table 3.** Values of non-dimensional arguments, predicted concentrations, uncertainty for phosphorus.

| Month | $C_{\text{measured}}$<br>kg m <sup>-3</sup> | $\pi_2$<br>– | $\pi_1$<br>– | $C_{\text{predicted}}$<br>kg m <sup>-3</sup> | $\sigma$<br>% |
|-------|---|--------------|--------------|--|---------------|
| 1     | $4.000 \times 10^{-5}$                      | 0.981988     | 0.155932     | $3.989 \times 10^{-5}$                       | 0.27          |
| 2     | $3.880 \times 10^{-5}$                      | 0.973917     | 0.139434     | $3.751 \times 10^{-5}$                       | 3.32          |
| 3     | $3.940 \times 10^{-5}$                      | 0.994492     | 0.271322     | $6.041 \times 10^{-5}$                       | 53.32         |
| 4     | $3.880 \times 10^{-5}$                      | 1.000356     | 0.231500     | $4.792 \times 10^{-5}$                       | 23.50         |
| 5     | $3.620 \times 10^{-5}$                      | 1.004842     | 0.276793     | $5.116 \times 10^{-5}$                       | 41.33         |
| 6     | $3.520 \times 10^{-5}$                      | 1.014955     | 0.162776     | $2.652 \times 10^{-5}$                       | 24.65         |
| 7     | $3.920 \times 10^{-5}$                      | 1.000950     | 0.178467     | $3.710 \times 10^{-5}$                       | 5.34          |
| 8     | $3.520 \times 10^{-5}$                      | 1.002628     | 0.168587     | $3.096 \times 10^{-5}$                       | 12.04         |
| 9     | $3.600 \times 10^{-5}$                      | 1.003621     | 0.148660     | $2.765 \times 10^{-5}$                       | 23.18         |
| 10    | $3.880 \times 10^{-5}$                      | 1.001043     | 0.218448     | $4.491 \times 10^{-5}$                       | 15.76         |
| 11    | $4.520 \times 10^{-5}$                      | 1.003698     | 0.189409     | $4.420 \times 10^{-5}$                       | 2.20          |
| 12    | $4.780 \times 10^{-5}$                      | 0.985081     | 0.114483     | $3.394 \times 10^{-5}$                       | 29.00         |

provides a way of generating sets of non-dimensional arguments (Zeleňáková and Čarnogurská, 2008; Buckingham, 1914).

It is clear that input data and selection of relevant parameters are the most important factors in predicting pollutant concentration in a water stream.

The choice of variables is influenced by the ability of an organisation to provide the facilities, and trained operators, to enable the selected measurements to be made accurately. Full selection of variables must be made in relation to assessment objectives and specific knowledge of each individual situation (Chapman, 1996).

The flow and velocity of a stream can significantly affect its ability to transport pollutants (Chapman, 1996). Thus measurement of velocity is extremely important in any modelling. It enables the prediction of movement of pollutants within water streams. Water bodies undergo temperature variations along with normal climatic variations which occur seasonally. The temperature of surface waters is influenced by latitude, altitude, time of day, time of year, the flow and depth of the water body. Temperature affects also

**Table 4.** Model performance.

| Indicator                  | Magnitude of error for N model | Magnitude of error for P model |
|----------------------------|--------------------------------|--------------------------------|
| RMSE (mg L <sup>-1</sup> ) | 0.000700                       | 0.0000142                      |
| RRMSE (%)                  | 35                             | 38                             |
| CRM                        | -7.96                          | -1                             |

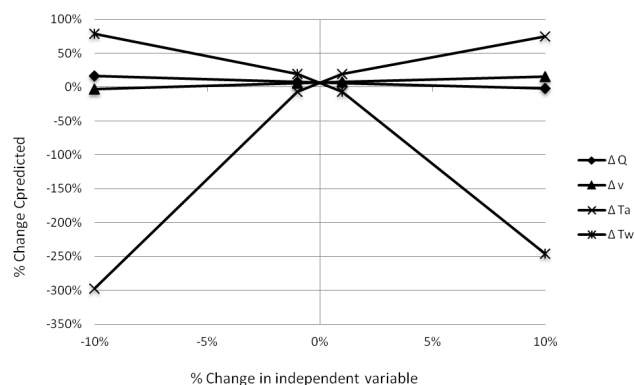
**Table 5.** Error magnitude considering only average values.

| Indicator                  | Value of error for N model | Value of error for P model |
|----------------------------|----------------------------|----------------------------|
| RMSE (mg L <sup>-1</sup> ) | 0.000142                   | 0.000001                   |
| RRMSE (%)                  | 7.4                        | 2.8                        |

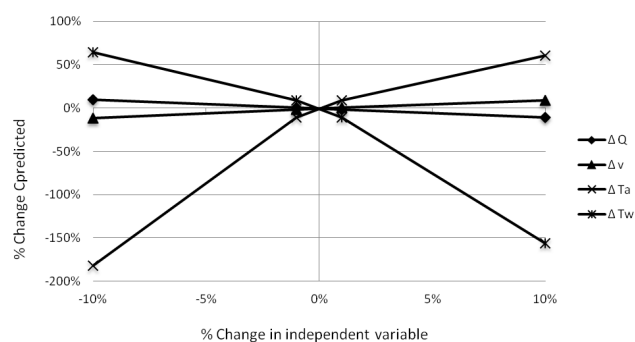
the physical, chemical and biological processes in rivers and, therefore, the values of many variables. The size of the catchment area controls the fluctuations in water level, velocity and discharge. The final selection of variables is made in relation to the assessment objectives.

The manuscript presents possible and instructive approach to determine pollutant concentrations in rivers using dimensional analysis if we know the selected variables. Modelling pollutant occurrence in a stream is a very complex problem. This issue is influenced by a lot of natural as well as artificial phenomena, particularly human activities in the catchment. To consider all the variables influencing this process is practically impossible. We chose the main ones to present the possibility of using dimensional analysis as a tool for description of a phenomenon as complex as pollutant occurrence in a stream.

The aim of the paper was not only scientific but also practical to use the results of research in water management practice. We intend to broaden our research and consider other equally important variables, e.g. rainfall or amount of nutrients applied in the catchment (to obtain these data is rather complex), but this will be the task of our future research. In this paper we would like to point out the broad applicability of this model. It could be used in any catchment and for any pollutant. The important point is to calculate new regression coefficients *A* and *B* for each catchment or each pollutant. So this work has general significance, not only for the field of water quality modelling. Its importance is for water management in general, to predict the pollutant concentration in a stream and to save a lot of expensive monitoring equipment and its maintenance as well as laboratory work and finally prediction of pollutant concentration in the river will be able to achieve good water status according to the Water Framework Directive. It is also necessary to acknowledge that the model has empirical basis, and is based on data coming from the period 1995 to 2002, it has empirical basis. If some changes are done on the watershed (more nutrient sources are added or



**Fig. 6.** Model sensitivity of nitrogen concentration to changes in input parameters ( $Q$  – flow,  $v$  – velocity of water in the stream,  $T_a$  – temperature of air,  $T_w$  – temperature of water).



**Fig. 7.** Model sensitivity of phosphorus concentration to changes in input parameters ( $Q$  – flow,  $v$  – velocity of water in the stream,  $T_a$  – temperature of air,  $T_w$  – temperature of water).

removed), the model cannot predict the consequences, since it needs new data to be calibrated. Maybe the model can predict the response of the system in terms of variation in stream velocity or flow, but not in terms of changing in the sources of nutrients.

Calculated uncertainty can be considered as an acceptable factor. Differences occur for a variety of reasons, e.g. rainfall, influence of source of pollution, outflow of waste water. The highest differences, as can be seen in Fig. 4 (measurement 42) could occur because of an error in taking a sample or in determining concentration. The differences between measured and predicted nitrogen and phosphorus concentrations can occur because the selection of relevant parameters did not involve all the effects which pollutant concentration depends on. The last reason may be that the measured values are not exactly stated.

## 5 Conclusions

Agriculture and urban activities as has been mentioned above are major sources of nitrogen and phosphorus in aquatic ecosystems. These non-point inputs of nutrients are difficult to measure and regulate because they derive from activities dispersed over wide areas of land and are variable in time due to effects of the weather. Nutrient enrichment seriously degrades aquatic ecosystems and impairs the use of water particularly for drinking, industry, agriculture and recreation (Carpenter et al., 1998).

Water quality modelling can be a valuable tool for water management since it can simulate the potential response of the aquatic system to such changes as the increase in nutrient levels. The use of generally available models should be verified with data obtained from the river for which its use is being considered.

River quality models seek to describe the spatial and temporal changes in constituents which are of concern. This paper presents the possibility of pollutant concentration prediction in a water stream. The developed model is based on dimensional analysis, which is applied in engineering and water management to understand physical situations (Čarnogurská, 2000).

The main objectives of the present research are to investigate options for estimating the parameters of the models and to develop a usable model for concentration prediction in a stream.

The variation of pollutant concentrations in surface waters stimulates broad interest among scientists and researchers in the field of water pollution control. Models are useful in defining the nature of water systems and the relations among their components. A model for the calculation of nitrogen and phosphorus concentrations has been developed and verified for the river station Ižkovce in River Laborec in eastern Slovakia. Based on the study results, it can be concluded that the water quality prediction model is applicable to this regulated large river for water quality management.

A sensitivity analysis carried out indicated the importance of the temperature of air and the temperature of water on the pollutant concentrations predicted by the model. The model performance evaluation showed the best results when average measured and predicted values were considered. Less robust results were found when only single values were used.

This approach could be used to calculate the parameters for other similar streams, if the coefficients in the equations were similar. Alternatively, further work would be needed to explore how these coefficients vary between streams.

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