Extended Water Allocation System Model Formulation

D. E. Rosenberg[1]

[1]{Department of Civil and Environmental Engineering and Utah Water Research Laboratory, Utah State University, Logan, Utah, United States}

Correspondence to: D. E. Rosenberg (david.rosenberg@usu.edu)

This supplemental material presents the complete optimization program for the extended Water Allocation System model in standard form: the objective function followed by nine primary and five additional constraints. Notation follows Fisher et al. (2005) and Rosenberg et al. (2008). Model extensions include and allow return flows from agriculture, brine waste from desalination, multiple water quality types to meet a minimum in-stream flow requirement, fixed-increment infrastructure capacity expansions, and are shown in constraints 1and 6 – 9.

Objective Function

$$
\begin{aligned}
Max\,Z = \sum_e prob_e \cdot \Bigg[& \sum_i \sum_d \frac{b_{ide}\left(\sum_q QD_{idqe}\right)^{\alpha_{ide}+1}}{\alpha_{ide}+1} - \sum_i \sum_q \sum_s cs_{iqse} QS_{iqse} - \sum_p \sum_{j \neq p} \sum_q ctr_{qpje} QTR_{qpje} \\
& - \sum_i \sum_d \sum_q cr_{idqe} QTW_{idqe} - \sum_i \sum_d ce_{ide}\left(\sum_q QDC_{idqe}\right) \Bigg] \\
& - \sum_p \sum_{j \neq p} \sum_q cxtr_{qpj} XTR_{qpj} - \sum_i \sum_q \sum_s cxs_{iqs} XS_{iqs} - \sum_i \sum_q \left(cxtw_{iq} XTW_{iq} + cxl_{iq} XL_{iq}\right) \\
& - \sum_i \sum_d cxcon_{id} XCON_{id}
\end{aligned}
$$

Subject to (constraints)

1. Continuity equation (mass balance) at each district for each quality in each event

$$
\sum_d QDC_{idqe} = \left(\sum_s QS_{iqse} + \sum_d QTW_{idqe} + QB_{iqe} + \sum_p QTR_{qpie} - \sum_p QTR_{qipe} \right) \cdot \left(1 - dl_{0iq} - XL_{iq}\right), \; \forall i,q,e
$$

2. Continuity equation (mass balance) at each node for each quality in each event

$$\sum_p QTR_{qpne} = \sum_p QTR_{qnpe}, \ \forall n, q, e$$

3. Treated waste-water comes from water demanded

$$\sum_q QTW_{idqe} = PR_{ide} \sum_q QDC_{idqe}, \ \forall i, d, e$$

4. Lower limit on demand for each water use sector in each district in each event

$$\sum_q QD_{idqe} \geq \left(\frac{p_{\max}}{b_{ide}} \right)^{\frac{1}{\alpha_{ide}}}, \ \forall i, d, e$$

5. User conservation reduces real water demanded without loss of economic benefit

$$QDC_{idqe} = QD_{idqe} \cdot \left(1 - pcon_{0\ id} - XCON_{id}\right), \ \forall i, d, q, e$$

6. Brine waste generated is a fraction of desalinated water

$$QB_{iqe} = \sum_{q_e \in dq_i(q)} bf_{iq_2} \cdot QS_{iq_2 se}, \ \forall i, q, e, s = desal$$

7. Supply expansions limited to fixed increments

$$XS_{iqs} = qs_{\text{int}erval\ iqs} \cdot XSLEV_{iqs}, \ \forall i, q, s$$

8. Conveyance expansions limited to fixed increments

$$XTR_{qpj} = qtr_{\text{int}erval\ \ qpj} \cdot XTRLEV_{iqs}, \ \forall q, p, j$$

9. Flows of one or more water quality types must meet the minimum required flow along a conveyance link either:

(a) absolutely in every water availability event

$$\sum_{q \in qt(p,j)} QTR_{qpje} \geq qtr_{\text{minmq}\ pj}, \ \forall p, j, e, \text{ or}$$

(b) on average so that the expected flow meets the minimum required flow

$$\sum_e \left(prob_e \cdot \sum_{q \in qt(p,j)} QTR_{qpje} \right) \geq qtr_{\text{minmq}\ pj}, \ \forall p, j.$$

With the following bounds

$$QS_{iqse} \le qs_{0\ iqs} + XS_{iqs},\ \forall i,q,s,e \qquad qs_{0\ iqs} + XS_{iqs} \le qs_{\max\ iqs},\ \forall i,q,s$$

$$PR_{ide} \le pr_{\max\ id},\ \forall i,d,e \qquad QS_{iqse} \le qs_{flow\,iqse},\ \forall i,q,s,e$$

$$QTR_{qpje} \le qtr_{0\ qpj} + XTR_{qpj},\ \forall q,p,j,e \qquad qtr_{0\ qpj} + XTR_{qpj} \le qtr_{\max\ qpj},\ \forall q,p,j$$

$$QTR_{qpje} \ge qtr_{\min\ qpj},\ \forall q,p,j,e \qquad pcon_{0\ id} + XCON_{id} \le pcon_{\max\ id},\ \forall i,d$$

$$\sum_{d} QTW_{idq} \le qtw_{0\ iq} + XTW_{iq},\ \forall i,q \qquad dl_{0\ iq} + XL_{iq} \le dl_{\max\ iq},\ \forall i,q$$

and all variables positive.

Variables are:

Z = net benefit from water in millions of dollars;

QB_{iqe} = brine waste volume generated in district *i* in event *e* of quality *q* in 10^6 m^3;

QS_{iqse} = volume supplied by source *s* of quality *q* in district *i* in event *e* in 10^6 m^3;

QD_{idqe} = volume of quality *q* demanded by sector *d* in district *i* in event *e* in 10^6 m^3;

QDC_{idqe} = volume demanded after conservation in 10^6 m^3;

QTR_{qpje} = volume of water quality *q* transferred from *p* to *j* in event *e* in 10^6 m^3;

QTW_{idqe} = sector *d* wastewater treated to quality *q* in district *i* in event *e* in 10^6 m^3;

PR_{ide} = percent of sector *d* wastewater treated in district *i* in event *e* in fraction;

XS_{iqs} = Size of supply expansions for source *s* of quality *q* in district *i* in 10^6 m^3;

$XSLEV_{i,q,s}$ = Number of source expansions implemented at district *i* of quality *q* for source *s* in integers [0, 1, 2, …];

XTR_{qpj} = Size of conveyance expansions from point *p* to *j* of quality *q* in10^6 m^3;

$XTRLEV_{qpj}$ = Number of conveyance expansions implemented of quality *q* from *p* to *j* in integers [0, 1, 2, …];

XTW_{id} = Size of wastewater reuse plant expansions in district *i* for quality *q* 10^6 m^3;

XL_{iq} = Leak reduction program expansion in district *i* for quality *q* in fraction;

$XCON_{id}$ = User conservation program expansion in district *i* for quality *q* in fraction;

Indices are:

p = points (districts and nodes);

i = district;

n = nodes;

d = water sector (urban, industrial, or agricultural);

s = supply source or step;

q = water quality type (fresh, recycled water);

e = events (water supply availability / demand)

Parameters are:

α_{ide} = exponent of inverse demand function for demand *d* in district *i* in event *e*;

b_{ide} = coefficient of inverse demand curve for demand *d* in district *i* in event *e*;

bf_{iq} = brine fraction that represents the volume of brine generated for each 1 m^3 of desalinated water produced [unitless];

ce_{ide} = unit environmental cost of water discharged by demand sector *d* in district *i* in event *e* in \$ m^{-3};

cr_{idqe} = unit cost to treat sector *d* waste in district *i* to quality *q* in event *e* in \$ m^{-3};

cs_{iqse} = unit cost to supply new water of quality *q* from source *s* in district *i* in event *e* in \$ m^{-3};

ctr_{qpje} = unit cost to transport water quality *q* from point *p* to *j* in event *e* in \$ m^{-3};

cxs_{iqs} = annualized capital cost to expand source *s* of quality *q* in district *i* in \$ m^{-3};

$cxtr_{iqs}$ = annualized capital cost to expand conveyance capacity of quality *q* from point *p* to *j* in \$ m^{-3};

$cxtw_{iq}$ = annualized capital cost to expand wastewater treatment capacity to quality *q* in district *i* in \$ m^{-3};

$cxcon_{id}$ = annualized capital cost to expand user conservation program in district *i* for sector *d* in \$ $fraction^{-1}$;

cxl_{iq} = annualized capital cost to expand leak reduction program in district *i* for quality *q* in \$ $fraction^{-1}$;

$dq_i(q)$ = set of source water quality types that, when desalinated in district *i*, generate brine of quality *q* [unitless];

p_e = probability of event *e* in fraction;

$p_{max\ id}$ = maximum price of water from demand sector *d* in district *i* in \$ m^{-3};

$pr_{max\ id}$ = maximum percent of waste from demand sector *d* in district *i* that can be treated in fraction;

$qs_{0\ iqs}$ = existing capacity of source *s* of quality *q* in district *i* in $10^6\ m^3$;

$qs_{flow\ iqse}$ = availability of source *s* of quality *q* in district *i* in event *e* in $10^6\ m^3$;

$qs_{interval\ iqs}$ = fixed interval to expand source capacity *s* of quality *q* in district *i* in $10^6 m^3$;

$qs_{max\ iqs}$ = maximum capacity for source *s* of quality *q* in district *i* in $10^6\ m^3$;

$qt(p,j)$ = set of water quality types whose flows can count towards the minimum required flow along the conveyance link from *p* to *j* [unitless];

$qtr_{0\ qpj}$ = existing conveyance capacity for quality *q* from point *p* to *j* in $10^6\ m^3$;

$qtr_{interval\ qpj}$ = fixed interval to expand conveyance capacity of quality *q* from *p* to *j* in $10^6 m^3$;

$qtr_{min\ qpj}$ = minimum required flow of quality *q* from point *p* to *j* in $10^6\ m^3$;

$qtr_{minmq\ pj}$ = minimum required flow from point *p* to *j* that multiple water quality types must satisfy on average in $10^6\ m^3$;

$qtw_{0\ iq}$ = existing capacity to treat water to quality *q* at district *i* in $10^6\ m^3$;

$pcon_{0\ id}$ = reduction in use associated with existing conservation programs for sector *d* in district *i* in fraction;

$pcon_{max\ id}$ = maximum reduction in use from conservation programs for sector *d* in district *i* in fraction;

$dl_{0\ iq}$ = existing leak rate for quality *q* in district *i* in fraction;

$dl_{max\ iq}$ = maximum reduction in leakage rate for quality *q* in district *i* in fraction;

Additional Constraints

10. Total demand consists of paid water and unaccounted-for losses

$$QDC_{idqe} = QD_{paid\ idqe} + r_{iqe}\, pr_{unpaid\ ide},\ \forall i, d, q, e\,.$$

11. Demand for certain water quality types must be less than a specified quantity

$$\sum_{d} QDC_{idqe} \le q_{rec\max\ iq},\ \forall i, q, e$$

12. Demand for certain water quality types must be less than a specified percentage of total demand.

$$\sum_{d} QDC_{idqe} \le p_{rec\max\ iq} \sum_{d,q2} QD_{id,q2,e},\ \forall i, q, e$$

13. Use from a pool of sources must be less than a specified quantity

$$\sum_{iqs} indcp_{ciqs} QS_{iqse} \le q_{shared\ ce},\ \forall c, e$$

14. Minimum required allocation to each sector

$$\sum_{q} QDC_{idqe} \ge q_{required\ ide},\ \forall i, d, e$$

References

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