Supplementary material for: Temporal dynamics of hydrological threshold events.

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This material provides full solutions to the raw and central moments of the first passage time statistics detailed in the above mentioned article and a figure showing the correspondence between analytically derived statistics and those obtained by numerical simulation.

1 Raw Moments

1.1 First raw moment: $T_1\left[s_0, s_\xi\right]$

$$T_1\left[s_0, s_\xi\right] = \frac{\alpha(e^{(\alpha-\beta)s_\xi}\alpha - e^{(\alpha-\beta)s_0}\beta)}{(\alpha-\beta)^2\lambda} - \frac{\beta(\alpha(s_\xi - s_0)+1)}{(\alpha-\beta)\lambda}$$

1.2 Second raw moment: $T_2\left[s_0, s_\xi\right]$

$$T_2\left[s_0, s_\xi\right] = (e^{-\beta s_\xi}(2e^{2\alpha s_\xi - \beta s_\xi}\alpha^4 - 2e^{\alpha s_0 - \beta s_0 + \alpha s_\xi}\alpha^3\beta + e^{\beta s_\xi}\beta^2(-\alpha+\beta)(2\beta - \alpha(4\beta + \alpha(-2+(\alpha-\beta)(s_0-s_\xi)))(s_0-s_\xi)) + 2e^{(\alpha-\beta)s_0+\beta s_\xi}\alpha\beta^2(\alpha - 3\beta + \alpha\beta s_0 - \beta^2 s_0 + \alpha(\alpha-\beta)s_\xi) - 2e^{\alpha s_\xi}\alpha^2\beta(-\alpha^2(s_0 - 2s_\xi) + \alpha(3+\beta s_0 - \beta s_\xi) - \beta(5+\beta s_\xi)))) \div ((\alpha-\beta)^4\lambda^2)$$

1.3 Third raw moment: $T_3\left[s_0, s_\xi\right]$

$$T_3\left[s_0, s_\xi\right] = (e^{-\beta(s_0+3s_\xi)}(6e^{\beta s_0 + 3\alpha s_\xi}\alpha^6 - 6e^{\alpha s_0 + 2\alpha s_\xi + \beta s_\xi}\alpha^5\beta - 3e^{\alpha s_0 + 3\beta s_\xi}\alpha\beta^3(\beta(\alpha^2\beta s_0^2 - 2\alpha(1+\beta s_0(3+\beta s_0)) + \beta(12+\beta s_0(6+\beta s_0))) + 2\alpha(\alpha-\beta)(\alpha + \alpha\beta s_0 - \beta(4+\beta s_0))s_\xi + \alpha^2(\alpha-\beta)^2 s_\xi^2) + 6e^{2\alpha s_\xi + \beta(s_0+s_\xi)}\alpha^4\beta(\alpha^2(s_0 - 3s_\xi) + \alpha(-5-\beta s_0 + \beta s_\xi) + \beta(9+2\beta s_\xi)) + e^{\beta(s_0+3s_\xi)}(\alpha-\beta)\beta^3(-6\beta(\alpha+\beta) + \alpha^3(\alpha-\beta)^2 s_0^3 + 6\alpha(\alpha-3\beta)\beta s_\xi - 3\alpha^2(\alpha-3\beta)(\alpha-\beta)s_\xi^2 - \alpha^3(\alpha-\beta)^2 s_\xi^3 - 3\alpha^2(\alpha-\beta)s_0^2(\alpha - 3\beta + \alpha(\alpha-\beta)s_\xi) + 3\alpha s_0(-2(\alpha - 3\beta)\beta + 2\alpha(\alpha-3\beta)(\alpha-\beta)s_\xi + \alpha^2(\alpha-\beta)^2 s_\xi^2)) + 6e^{\alpha s_0 + \alpha s_\xi + 2\beta s_\xi}\alpha^3\beta^2(2\alpha^2 s_\xi + \alpha(3 + \beta s_0 - \beta s_\xi) - \beta(7+\beta(s_0+s_\xi))) + 3e^{\beta s_0 + \alpha s_\xi + 2\beta s_\xi}\alpha^2\beta^2(\alpha^4(s_0 - 2s_\xi)^2 - 2\alpha^3(s_0 -$$

$$2s_\xi)(3 + \beta s_0 - \beta s_\xi) + 2\alpha\beta(-13 + \beta s_\xi(5 + \beta s_\xi) - \beta s_0(6 + \beta s_\xi)) + \beta^2(30 + \beta s_\xi(10 + \beta s_\xi)) + \alpha^2(6 + \beta(18s_0 + \beta s_0^2 - s_\xi(32 + 3\beta s_\xi)))))) \div ((\alpha - \beta)^6 \lambda^3)$$

1.4 Fourth raw moment: $T_4\left[s_0, s_\xi\right]$

$$T_4\left[s_0, s_\xi\right] = (e^{-\beta s_0-\alpha s_\xi-7\beta s_\xi}(24e^{\beta s_0+5\alpha s_\xi+3\beta s_\xi}\alpha^8 - 24e^{\alpha s_0+4(\alpha+\beta)s_\xi}\alpha^7\beta + 24e^{\alpha s_0+3\alpha s_\xi+5\beta s_\xi}\alpha^5\beta^2(3\alpha^2 s_\xi + \alpha(5 + \beta s_0 - \beta s_\xi) - \beta(11 + \beta s_0 + 2\beta s_\xi)) + 24e^{\beta s_0+4(\alpha+\beta)s_\xi}\alpha^6\beta(\alpha^2(s_0 - 4s_\xi) + \alpha(-7 - \beta s_0 + \beta s_\xi) + \beta(13 + 3\beta s_\xi)) + e^{(\alpha+7\beta)s_\xi}\beta^4(4e^{\alpha s_0}\alpha(\beta(\alpha^3\beta s_0^2(-3 + \beta s_0) - 3\alpha^2(-2 + \beta s_0(-2 + \beta s_0)(3 + \beta s_0)) + 3\alpha\beta(-10 + \beta s_0(2 + \beta s_0)(3 + \beta s_0)) - \beta^2(60 + \beta s_0(36 + \beta s_0(9 + \beta s_0)))) + 3\alpha(\alpha - \beta)\beta(\alpha^2\beta s_0^2 - 2\alpha(1+\beta s_0(4+\beta s_0)) + \beta(20+\beta s_0(8+\beta s_0)))s_\xi + 3\alpha^2(\alpha-\beta)^2(\alpha+\alpha\beta s_0 - \beta(5+\beta s_0))s_\xi^2 + \alpha^3(\alpha-\beta)^3 s_\xi^3) + e^{\beta s_0}(\alpha-\beta)(-24\beta(\alpha^2+3\alpha\beta+\beta^2) + \alpha^4(\alpha-\beta)^3 s_0^4 + 24\alpha\beta(\alpha^2-2\alpha\beta-4\beta^2)s_\xi - 12\alpha^2(\alpha-6\beta)(\alpha-\beta)\beta s_\xi^2 + 4\alpha^3(\alpha-4\beta)(\alpha-\beta)^2 s_\xi^3 + \alpha^4(\alpha-\beta)^3 s_\xi^4 - 4\alpha^3(\alpha-\beta)^2 s_0^3(\alpha-4\beta+\alpha(\alpha-\beta)s_\xi) + 6\alpha^2(\alpha-\beta)s_0^2(-2(\alpha-6\beta)\beta + 2\alpha(\alpha-4\beta)(\alpha-\beta)s_\xi + \alpha^2(\alpha-\beta)^2 s_\xi^2) + 4\alpha s_0(6\beta(-\alpha^2+2\alpha\beta+4\beta^2) + 6\alpha(\alpha-6\beta)(\alpha-\beta)\beta s_\xi - 3\alpha^2(\alpha-4\beta)(\alpha-\beta)^2 s_\xi^2 - \alpha^3(\alpha-\beta)^3 s_\xi^3))) - 12e^{\alpha s_0+2\alpha s_\xi+6\beta s_\xi}\alpha^3\beta^3(4\alpha^4 s_\xi^2 + 4\alpha^3 s_\xi(3+\beta s_0 - \beta s_\xi) + \beta^2(56 + \beta(s_0+s_\xi)(14 + \beta(s_0+s_\xi))) - 2\alpha\beta(17 + \beta(9s_0 + \beta s_0^2 - s_\xi(7+\beta s_\xi))) + \alpha^2(6+\beta(\beta s_0^2+s_0(4-6\beta s_\xi)-s_\xi(40+3\beta s_\xi)))) + 12e^{\beta s_0+3\alpha s_\xi+5\beta s_\xi}\alpha^4\beta^2(\alpha^4(s_0-3s_\xi)^2 - 2\alpha^3(s_0-3s_\xi)(5+\beta s_0-\beta s_\xi) + 2\alpha\beta(-41+2\beta s_\xi(2+\beta s_\xi) - 2\beta s_0(5+\beta s_\xi)) + 2\beta^2(45 + 2\beta s_\xi(9+\beta s_\xi)) + \alpha^2(20 + \beta(\beta s_0^2 + 2s_0(15+\beta s_\xi) - s_\xi(74+11\beta s_\xi)))) + 4e^{\beta s_0+2\alpha s_\xi+6\beta s_\xi}\alpha^2\beta^3(\alpha^6(s_0-2s_\xi)^3 - 3\alpha^5(s_0-2s_\xi)^2(3+\beta s_0-\beta s_\xi) + 3\alpha\beta^2(-56+\beta s_\xi(5+\beta s_\xi)(8+\beta s_\xi) - \beta s_0(42+\beta s_\xi(12+\beta s_\xi))) + \beta^3(210 + \beta s_\xi(90+\beta s_\xi(15+\beta s_\xi))) + 3\alpha^4(s_0-2s_\xi)(6+\beta(\beta s_0^2+s_0(13-\beta s_\xi)-s_\xi(22+\beta s_\xi))) + 3\alpha^2\beta(16+\beta(\beta s_0^2(7+\beta s_\xi) + s_0(72-\beta^2 s_\xi^2) - s_\xi(128+\beta s_\xi(21+\beta s_\xi)))) - \alpha^3(6+\beta(\beta^2 s_0^3+3\beta s_0^2(17+\beta s_\xi) - 3s_0(-36+\beta s_\xi(48+5\beta s_\xi)) + s_\xi(-210+\beta s_\xi(87+11\beta s_\xi))))))) \div (\lambda^4(\alpha-\beta)^8)$$

2 Correspondence between analytical and numerical solutions

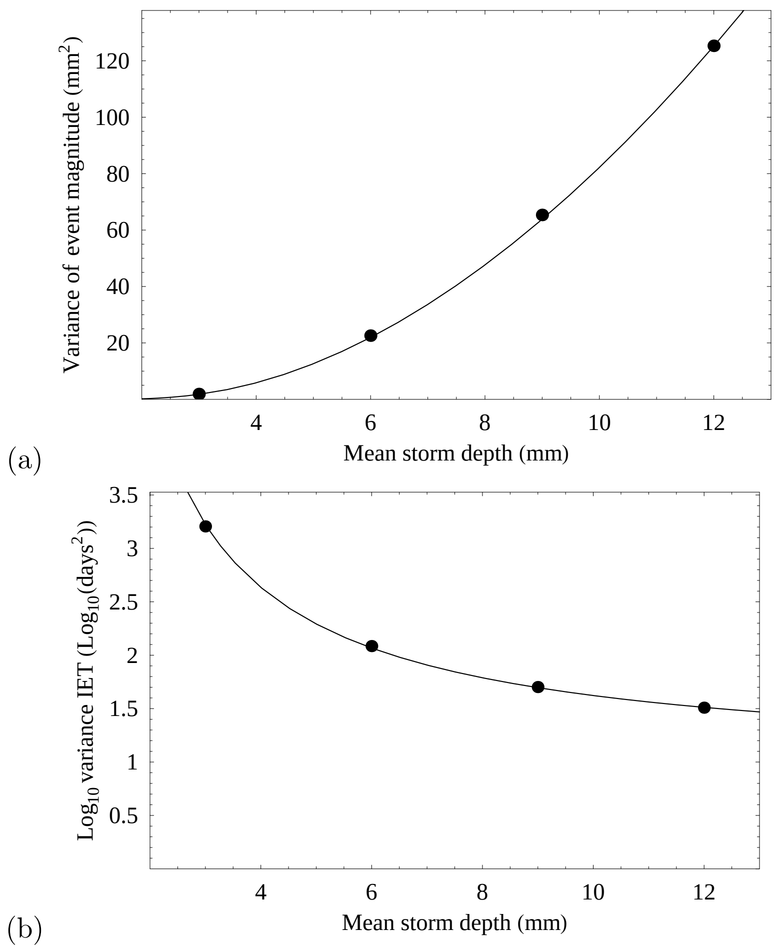

Figure 1: Correspondence between analytical (line) and numerical (points) of the variance of the saturation excess (a) event magnitude; and (b) inter-event time (IET); as a function of mean storm depth. Parameters used in the simulations include: $w_0 = 10$ mm, $\bar{t}_b = 3$ d, and $E_m = 2$ mm d^{-1}.